

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE - SEMESTER-IV • EXAMINATION – SUMMER 2013****Subject Code: 140001****Date: 05-06-2013****Subject Name: Mathematics - IV****Time: 10.30 am - 01.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) Find the real and imaginary part of  $f(z) = z^2 + 3z$ . 02
- (b) Sketch the region  $|z| \leq 1$ . Is it a domain? 02
- (c) For the principle branch show that  $\text{Log}(i^3) \neq 3 \text{Log}(i)$ . 02
- (d) Evaluate  $\oint_C (z^2 + 3) dz$  where C is any closed contour. Justify your answer. 02
- (e) Prove that  $\sin^{-1} z = -i \ln(iz + \sqrt{1 - z^2})$  02
- (f) Prove that  $E = 1 + \Delta$  where  $\Delta$  is forward deference and E is shift operator. 02
- (g) Discuss the singularity of the point  $z = 0$  for the function  $\frac{\sin z}{z}$ . 02

- Q.2**
- (a) Explain Newton's method for solving equation  $f(x) = 0$ . Apply this method to find the approximate solution of  $x^3 + x - 1 = 0$  correct up to three decimal. 07
- (b) Write the trapezoidal rule for numerical integration. Using Simpson's 1/3 rule evaluate  $\int_1^{2.5} f(x) dx$  from the following data. Take  $h = 0.3$ . 07

x	1	1.3	1.6	1.9	2.2	2.5
f(x)	1	1.69	2.56	3.61	4.84	6.25

**OR**

- (b) Write formula for Range Kutta method for order four. Apply Euler's method to find the approximate solution of  $\frac{dy}{dx} = x + y$  with  $y(0) = 0$  and  $h = 2$ . Show your calculation up to five iterations. 07
- Q.3** (a) Explain quadratic Lagrange interpolation. Compute  $f(9.2)$  by using Lagrange interpolation method from the following data. 07

x	9	9.5	11
f(x)	2.1972	2.2513	2.3979

- (b) Use Newton's forward deference method to find the approximate value of  $f(1.3)$  from the following data 07

x	1	2	3	4
F(x)	1.1	4.2	9.3	16.4

**OR**

- Q.3** (a) Write a formula for divided difference  $f[x_0, x_1]$  and  $f[x_0, x_1, x_2]$ . Using Newton's divided difference formula compute  $f(10.5)$  from the following data 07

x	10	11	13	17
F(x)	2.3026	2.3979	2.5649	2.8332

- (b) Use Gauss Seidel method to determine roots of the following simultaneous equations. 07

$$\begin{aligned} 2x - y &= 3 \\ x + 2y + z &= 3 \\ -x + z &= 3 \end{aligned}$$

- Q.4 (a)** Define a harmonic function. Show that  $u(x, y) = x^2 - y^2$  is harmonic. Find the corresponding analytic function  $f(z) = u(x,y) + iv(x,y)$ . **07**
- (b)** Define a linear fractional transformation (*Möbius* transformation). Find the bilinear transformation that maps the points  $z_1 = -1$ ,  $z_2 = 0$ ,  $z_3 = 1$  onto  $w_1 = -i$ ,  $w_2 = 1$ ,  $w_3 = i$  respectively. **07**  
Also find  $w$  for  $z = \infty$ .

**OR**

- Q.4 (a)** State de Moivre's formula. Find and graph all fifth root of unity in complex plane. **07**
- (b)** State Liouville's theorem and Maximum Modulus theorem. Without using integration show that  $\left| \oint_C \frac{e^z}{z+1} dz \right| \leq \frac{8\pi e^4}{3}$  where  $C$  is  $|z| = 4$  **07**

- Q.5 (a)** Evaluate (i)  $\int_0^{2+i} z^2 dz$  along the line  $y = x/2$  **07**
- (ii)  $\oint_C \frac{5z+7}{z^2+2z-3} dz$  where  $C$  is  $|z-2| = 2$
- (b)** Find Laurent's series expansion in power of  $z$  that represent  $f(z) = \frac{1}{z^2(1-z)}$  for domain (i)  $|z| < 1$  and (ii)  $|z| > 1$  **07**

**OR**

- Q.5 (a)** (i) Evaluate  $\oint_C \tan z dz$  where  $C$  is  $|z| = 2$  **07**
- (ii) Evaluate  $\oint_C \frac{2z+6}{z^2+4} dz$   $C$  is  $|z-i| = 2$
- (b)** Evaluate a real integral  $\int_0^{2\pi} \frac{1}{(2+\cos \theta)^2} d\theta$  using residue. **07**

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