Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV • EXAMINATION – SUMMER • 2014

Subject Code: 140001 **Date: 12-06-2014**

Subject Name: Mathematics - IV

Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Show that $\cos h^{-1} z = \ln(z+\sqrt{z^2-1})$ **02**

(b) Prove that the n th roots of unity are in geometric progression. **02**

(c) If $w = f(z) = 2iz + 6\bar{z}$, find u and v. Also find the value of f at $z = \frac{1}{2} + 4i$. **02**

(d) Define Domain. Is the set $|z-1+2i| \le 2$ domain ? **02**

(e) What does conformal mapping mean ? At what points is the mapping by $w = z^2 + \frac{1}{z^2}$ not conformal ? **03**

(f) Define Harmonic function. Show that $u = \frac{x}{x^2+y^2}$ is harmonic function for $R^2 - (0,0)$. **03**

Q.2 (a) (1) What is an analytic function ? Show that $f(z) = z^3$ is analytic everywhere. **04**

(2)Determine the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. **03**

(b) (1) Find analytic function $f(z) = u + iv$ *if* $u = x^3 - 3xy$. **04**

(2) Find and sketch the image of the region $|z| > 1$ under the transformation $w = 4z$. **03**

OR

(b) (1) Prove that $\int_c \frac{dz}{z-a} = 2\pi i$ and **04**

$\int_c (z-a)^n dz = 0$ [n, any integer # -1]

Where C is the circle $|z-a| = r$.

(2) Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$. **03**

Q.3 (a) Evaluate $\int_c \operatorname{Re} z dz$, where c is the shortest path from 1+i to 3+2i. **05**

(b) Evaluate $\oint_c \frac{dz}{z^2+1}$, where c is the $|z+i| = 1$, counterclockwise. **05**

(c) (1) Show that $f(z) = \bar{z}$ is nowhere differentiable. **02**

(2) Separate real and imaginary parts of sinhz . **02**

OR

Q.3 (a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $|z| < 1$. (ii) $1 < |z| < 2$. **05**

(b) Determine the poles of the function $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\int_c f(z)dz$, where C is the circle $|z|=2.5$. **05**

(c) (1) Find an upper bound for the absolute value of the integral $\int_C z^2 dz$, where C is the straight line segment from 0 to 1+i. **02**

(2) Prove (i) $E=e^{hD}$ **02**
(ii) $\Delta=E-1$

Q.4 **(a)** Using Newton –Raphson method, find a root of the equation $x^3+x-1=0$ correct to four decimal places. **05**

(b) Solve $x=\cos x$ by Bisection method correct to two decimal places. **05**

(c) Solve the following system of linear equations by Gauss elimination **04**

$$8y+2z=-7$$
$$3x+5y+2z=8$$
$$6x+2y+8z=26$$

OR

Q.4 **(a)** Find the positive solution of $f(x)=x-2\sin x=0$ by the secant method, starting from $x_0=2, x_1=1.9$. **05**

(b) Evaluate f(9),using Lagrange's interpolation and Newton's divided difference from the following data **05**

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

(c) Determine the largest eigen value of $A=\begin{bmatrix}1 & -3 & 2\\ 4 & 4 & -1\\ 6 & 3 & 5\end{bmatrix}$ by Power method. **04**

Q.5 **(a)** Using Newton's forward interpolation formula ,find the value of f(218), if **05**

x	100	150	200	250	300	350	400
f(x)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Trapezoidal rule (ii) Simpson's 1/3 rule taking h=1. **05**

(c) Using Euler's method , find an approximate value of y corresponding to x=1 given that $\frac{dy}{dx}=x+y$ and y=1 when x=0. **04**

OR

Q.5 **(a)** Evaluate $\int_0^3 \frac{1}{1+x}dx$ with n=6 by using Simpson's 3/8 rule and hence calculate ln2. Estimate the bound of error involved in the process. **05**

(b) Apply Runge –kutta fourth order method, to find an approximate value of y when x=0.2 in steps of 0.1,if $\frac{dy}{dx}=x+y^2$,given that y=1 when x=0. **05**

(c) Solve , by Gauss-seidal iteration method ,the equations **04**

$$20x+y-2z=17$$
$$3x+20y-z=-18 \quad .$$
$$2x-3y+20z=25$$

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