

GUJARAT TECHNOLOGICAL UNIVERSITY
B. E. - SEMESTER – VII • EXAMINATION – WINTER 2012

Subject code: 172503**Date: 01/01/2013****Subject Name: Optimization Methods****Time: 10.30 am - 01.00 pm****Total Marks: 70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Read questions carefully.

Q.1 Do as Directed:**14**

- (i) Constraints of One of the LP Problem are $x_1 + x_2 \leq 10$ and $x_1 = 6$, what can be the geometric shape of feasible solution region?
- (ii) “Surplus variable is always accompanied by Artificial variable but the reverse may not be always true” Justify the Statement.
- (iii) By observing the coefficient of an Artificial Variable in the objective function, can you state the nature of objective function? Justify your answer.
- (iv) Constraints of One of the LP problem are $2x_1 + 3x_2 \leq 40$, $3x_1 + 2x_2 \leq 60$, $x_1 + x_2 \leq 25$, $x_1, x_2 \geq 0$, out of given constraints which constraint can be considered as redundant? Why?
- (v) “ Hungarian Method for Solving Assignment Problem was invented by renowned Mathematician Prof. Hungary.” Do you agree with the statement?
- (vi) If you solve a balanced transportation problem with 5 sources and 4 destinations with the help of simplex method, how many artificial variables will be there in the base of initial simplex tableau?
- (vii) One of the constraints of an LP Problem is $5x_1 - 4x_2 \leq 20$, as a student of) Optimization Methods, can you state practical relevance of coefficient of x_2 in given constraint?

Q.2 (a) A Dealer of Machine tools wishes to stock up his lot to maximize his profit. He can select Milling, Lathe and Shaping Machines which are valued on wholesale at Rs. 75000/-, Rs. 50000/- and Rs. 40000/- respectively and can be sold at Rs. 90000/-, Rs. 60000/- and Rs. 50000/- respectively per unit. For each type of Machine tools, the Probability of Sale are:

07

Type of M/C Tool:	Milling	Lathe	Shaping
Probability Of Sale:	0.7	0.8	0.6

For every two lathes, he must buy one milling or one shaping machine. He has Rs. 10,000,00/- to invest. Dealer is equipped with the Software for solving LPP and arrive at optimum solution but unable to formulate the said situation. As an Optimization Methods student, he wants expert advice from

your side. Help him to formulate the said situation.

- (b) In the modification of a plant layout of a factory, four new machines- Milling, Broaching, Grinding & Shaping are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limitation of space, Broaching machine and Grinding machine cannot be installed at C and A respectively. The cost of locating of machine i to place j , in rupees is shown as below: 07

	A	B	C	D	E
Milling	8900	9100	9500	9000	9100
Broaching	9200	8900	-	9000	8900
Grinding	-	9100	9400	9100	8700
Shaping	9400	8800	9200	8700	8800

Determine optimum assignment.

OR

- (b) The Owner of a small machine shop has four machinists available to assign to jobs for the day. Five jobs are offered with an expected profit in rupees for each machinist on each job as follows: 07

	J ₁	J ₂	J ₃	J ₄	J ₅
M ₁	62	78	50	101	82
M ₂	71	84	61	73	59
M ₃	87	92	111	71	81
M ₄	48	64	87	77	80

Determine the assignment of machinists to jobs that will result in a maximum profit. Which job should be declined?

- Q.3** (a) A steel company has three open hearth furnaces and five rolling mills. Transportation cost (Rs. Per Quintal) for shipping steel from furnaces to rolling mills are shown in the following table: 07

	RM ₁	RM ₂	RM ₃	RM ₄	RM ₅	Supply
F ₁	4	2	3	2	6	8
F ₂	5	4	5	2	1	12
F ₃	6	5	4	7	7	14
Demand	4	4	6	8	8	

Determine the Optimum Shipping Schedule.

- (b) Customers arrive at the First Class Ticket counter of a Theatre at a rate of 12 per hour. There is one clerk serving the customers at a rate of 24 per hour. 07
1. What is the probability that there is no customer at the counter?
 2. What is the probability that there are more than 2 customers at the counter?
 3. What is the probability that there is no customer waiting to be served?

OR

- Q.3 (a)** Following are transportation costs of shipping goods from source i to Destination j :

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	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	15	18	22	16	30
S ₂	15	19	20	14	40
S ₃	13	16	23	17	30
Demand	20	20	25	35	

Determine the Optimum Solution. Is the optimal solution obtained by you a unique one? If not, why?

- (b)** Assume that at a bank teller window, the customers arrive at the average rate of 20 per hour and the bank teller spends an average of 2 minutes per customer to complete a service. Customers, who arrive from an infinite population, are served on a first come first served basis.

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1. What is the expected waiting time in the system per customer?
2. What is the mean no. of customers waiting in the system?
3. What is the probability of zero customers in the system?

- Q.4 (a)** Convert the following game problem into a linear programming problem.

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	Player B		
Player A	8	20	-3
	6	25	4
	0	-8	12

- (b)** A bakery keeps stock of a popular brand of cakes. Previous experience shows the daily demand pattern for the item with associated probabilities, as given:

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Daily Demand(No.s): 0 10 20 30 40 50

Probability: .01 .20 .15 .50 .12 .02

Use the following sequence of random numbers to simulate the demand for next 10 days. Also find out the average demand per day.

Random numbers: 25, 39, 65, 76, 12, 05, 73, 89, 19, 49

OR

- Q.4 (a)** In respect of a two person zero sum game, the following pay off matrix is given:

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	Player B		
Player A	5	-2	-2
	-2	-2	2
	-2	1	-2

1. show that the above game has no saddle point
2. Show that the game cannot be reduced by applying dominance rule.
3. Obtain the value of game.

- (b) A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below: **07**
- | | | | | | | | | |
|---------------|----|---|----|----|----|----|------------|---|
| Demand/ Week: | 0 | 5 | 10 | 15 | 20 | 25 | Frequency: | 2 |
| | 11 | 8 | 21 | 5 | 3 | | | |
- Using the following sequence of random numbers, generate the demand for next 10 weeks. Also find out the average demand per week.
 Random numbers: 35, 52, 90, 13, 23, 73, 34, 57, 35, 83

- Q.5** Using two phase method OR Big M method, solve the following LP problem: **14**

Minimize $Z = 20X_1 + 10X_2$
 S.T. $X_1 + 2X_2 \leq 40$; $3X_1 + X_2 = 30$
 $4X_1 + 3X_2 \geq 60$; $X_1, X_2 \geq 0$

Also write dual of the given LP problem.

OR

- Q.5** Solve the following LP Problem using simplex method and answer the questions regarding a firm that manufactures machines X_1 and X_2 . **14**

Maximize $Z = 30X_1 + 80X_2$
 S.T. $2X_1 + 4X_2 \leq 1000$ Labour Hours constraint
 $6X_1 + 2X_2 \leq 1200$ Raw Material constraint
 $X_2 \leq 200$ Engine availability for machine X_2
 $X_1, X_2 \geq 0$

1. What is the best product mix and optimal profit?
2. What are shadow prices of the resources?
3. Which resource has highest marginal value?
4. Over what range in each of the RHS values are these shadows valid?

What are the range over which the objective function coefficients can vary for each of the two decision variables?
