

GUJARAT TECHNOLOGICAL UNIVERSITY**BE SEMESTER– 7th EXAMINATION – SUMMER 2015****Subject code: 172503****Date: 12/05/2015****Subject Name: Optimization Methods****Time: 02.30PM-05.00PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Solve the following L.P. problem using simplex method: **07**Maximize $Z = 5x_1 + 3x_2$

Subject to the constraints

$$x_1 + x_2 \leq 2,$$

$$5x_1 + 2x_2 \leq 10,$$

$$3x_1 + 8x_2 \leq 12,$$

And $x_1, x_2 \geq 0$.

- (b)** Five different machines can do any of the five required jobs, with different profit resulting from each assignment as shown in the table. Find out maximum profit possible through optimal assignment. **07**

Jobs	Machine				
	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

- Q.2 (a)** A manufacturing company has three plants A, B, C which supply to the distributors located at P, Q, R, S, T. Monthly plant capacities are 80, 50 and 90 units respectively. Monthly requirements of distributors are 40, 40, 50, 40, and 80 units respectively. Unit transportation costs are given below in rupees: **07**

From	To				
	P	Q	R	S	T
A	5	8	6	6	3
B	4	7	7	6	6
C	8	4	6	6	3

Determine the optimal distribution for the company in order to minimize total transportation cost.

- (b)** Solve the following L.P. problem using Big-M method: **07**

Minimize $Z = 3x_1 + 8x_2$

Subject to the constraints

$$x_1 + x_2 = 200,$$

$$x_1 \leq 80,$$

$$x_2 \geq 60,$$

And $x_1, x_2 \geq 0$.

OR

- (b) An automobile company has three major departments for manufacturing of two models A and B. The monthly capacities are given as below: **07**

	Per unit time requirements (hours)		Hours available this Month
	Model A	Model B	
Department-I	4.0	2.0	1600
Department-II	2.5	1.0	1200
Department-III	4.5	1.5	1600

The marginal profit per unit from of model A is Rs.400 and that of model B is Rs.100. Assuming that a company can sell any quantity of either product due to favorable market conditions, determine the optimum output for both the models, the highest possible profit for this month and the slack time in the three departments.

- Q.3 (a)** A company wants to produce three products: A, B, C. Unit contributions of the products are Rs.5, 10, 8 respectively. Each unit of product A requires 3 Kg. of material, 4 machine hours and 2 labour hours; each unit of product B requires 5 Kg. of material, 4 machine hours and 4 labour hours and each unit of product C requires 2 Kg. of material, 4 machine hours and 5 labour hours. **10**

Everyday 60 Kg. of material, 72 machine hours and 100 labour hours are available. Find out the best production strategy. Investigate the effect of the solution of the following:

- (i) An increase of 10 machine hours,
 - (ii) A decrease of 5 Kg. of material,
 - (iii) 5 units of product A are to be produced.
- (b) Explain the scope and methodology of OR. **04**

OR

- Q.3 (a)** Using Graphical Method obtained the optimal strategies for both players and the values of the game for two person zero sum game whose pay-off matrix is given as follows: **07**

Player A	Player B	
	B ₁	B ₂
A ₁	-6	7
A ₂	4	-5
A ₃	-1	-2
A ₄	-2	5
A ₅	7	-6

- (b) Define: (i) Competitive game, (ii) Payoff matrix, (iii) Pure strategies, (iv) Mixed strategies, (v) Saddle point, (vi) Optimal strategies and (vii) Two person zero sum game. **07**

- Q.4 (a)** Arrivals at telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of phone call is assumed to be distributed exponentially, with mean 4 minutes. **07**

- (i) What is the probability that a person arriving at the booth will have to wait?
- (ii) The telephone department will install a second booth when convinced that an arrival would expect waiting for at least 4

minutes for a phone call. By how much should the flow of arrivals increase in order to justify a second booth?

(iii) What is the average length of queue that forms from time to time?

(iv) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

- (b) A salesman has to visit five cities A, B, C, D and E. The distance (in hundred km) between the five cities are as follows: **07**

		To City				
		A	B	C	D	E
From City	A	-	17	16	18	14
	B	17	-	18	15	16
	C	16	18	-	19	17
	D	18	15	19	-	18
	E	14	16	17	18	-

If the salesman starts from city A and has to come back to city A, which route should he select so that total distance travelled by him is minimized?

OR

- Q.4 (a)** A bank has one drive-in counter. It is estimated that cars arrive **07**

according to Poisson distribution at the rate of 2 every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arriving cars can wait outside this space, if necessary. It takes 1.5 minutes on an average to serve a customer, but the service time actually varies according to an exponential distribution. You are required to find:

- The probability of time, the facility remains idle;
- The expected number of customers waiting but currently not being served at a particular point of time;
- The expected time a customer spends in the system, and
- The probability that waiting line will exceed the capacity of the space leading to the drive-in counter.

- (b) A company has four warehouses a, b, c, and d. It is required to deliver a product from these warehouses to their customers A, B and C. The warehouses have the following amounts in stock: **07**

Warehouse	a	b	c	d
No of Units	15	16	12	13

And the customer's requirements are:

Customer	A	B	C
No of Units	18	20	18

The table below shows the costs of transporting one unit from warehouse to customer:

		Warehouse			
		a	b	c	d
Customer	A	8	9	6	3
	B	6	11	5	10
	C	3	8	7	9

Find the optimal transportation routes.

- Q.5 (a)** A hospital administrator has compiled statistic relating to the patients arrival pattern and service pattern as given below. He has also decided **07**

to evaluate the operations by using the simulations technique.

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Service time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:

- (i) The hospital starts at 8.00 hours,
- (ii) Only one clinic is set up,
- (iii) The following 10 random numbers are to be used depict patients arrival pattern: 78, 26, 94, 08, 46, 63, 18, 35, 59 and 12.
- (iv) The following 10 random numbers are to be used depict the service pattern: 44, 21, 73, 96, 63, 35, 67, 31, 84 and 24.

(b) Write the dual of the following primal problem: 04

Minimize $Z_x = 18x_1 + 10x_2 + 11x_3$

Subject to the constraints

$$4x_1 + 6x_2 + 5x_3 \geq 480,$$

$$12x_1 + 10x_2 + 10x_3 \geq 1200,$$

$$10x_1 + 15x_2 + 7x_3 \leq 1500,$$

$$x_2 \geq 50,$$

$$x_1 - x_2 \leq 0,$$

And $x_1, x_2, x_3 \geq 0$.

(c) What do you understand by (i) Queue discipline, (ii) Arrival process and (iii) Service process. 03

OR

Q.5 (a) Consider the following transportation problem (cost is in Rs.). Find an Optimal solution using "Stepping Stone Method". 07

Depot \ Factory	D	E	F	G	Factory capacity
A	4	6	8	6	700
B	3	5	2	5	400
C	3	9	6	5	600
Depot Requirement	400	450	350	500	1700

(b) Define simulation. What are the advantages and limitations of simulation model? 07
