

Seat No: \_\_\_\_\_

Enrolment No: \_\_\_\_\_

# Gujarat Technological University

DIPLOMA ENGINEERING – SEMESTER – 1 • EXAMINATION – SUMMER- 2015

Subject Code: 3300001

Date: 26/05 /2015

Subject Name: Basic Mathematics

Time:02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q:1 Fill in the blanks using appropriate choice from the given options. 14

- 1  $\log_2 8 =$  \_\_\_\_\_  
(a) 3 (b) 0 (c) 1 (d) 2
- 2  $4^{\log_4 2} =$  \_\_\_\_\_  
(a) 0 (b) 4 (c) 2 (d) 1
- 3  $\log_3 2 \times \log_2 3 =$  \_\_\_\_\_  
(a) 3 (b) 1 (c)  $\log_2 5$  (d)  $\log_3 5$
- 4 If  $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$  then  $x =$  \_\_\_\_\_  
(a) -2 (b) 2 (c) -4 (d) 4
- 5 Order of  $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$  is \_\_\_\_\_  
(a) 2x3 (b) 2x2 (c) 3x2 (d) 3x3
- 6 If  $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$  then  $A^T =$  \_\_\_\_\_  
(a)  $A^{-1}$  (b) A (c)  $\begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 4 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$
- 7  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} =$  \_\_\_\_\_  
(a)  $\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$  (b)  $\begin{bmatrix} 5 & 3 \\ 9 & 7 \end{bmatrix}$  (c)  $\begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$  (d) none of these

- 8  $\sin\left(\frac{\pi}{2} - \theta\right) = \underline{\hspace{2cm}}$   
 (a)  $-\cos\theta$                       (b)  $\cos\theta$                       (c)  $\sin\theta$                       (d)  $-\sin\theta$
- 9  $\sin^{-1} x + \cos^{-1} x = \underline{\hspace{2cm}}$   
 (a) 0                      (b) 1                      (c)  $\pi$                       (d)  $\frac{\pi}{2}$
- 10 Period of  $\sin 2x$  is  $\underline{\hspace{2cm}}$   
 (a)  $2\pi$                       (b)  $\frac{\pi}{2}$                       (c)  $\pi$                       (d) 2
- 11  $\sin 2\theta = \underline{\hspace{2cm}}$   
 (a)  $2\sin\theta$                       (b)  $2\sin\theta\cos\theta$                       (c)  $\sin^2\theta - \cos^2\theta$                       (d)  $\cos^2\theta - \sin^2\theta$
- 12 Volume of sphere of radius 'r' is  $\underline{\hspace{2cm}}$   
 (a)  $\frac{4}{3}\pi r^3$                       (b)  $\frac{1}{3}\pi r^3$                       (c)  $\frac{3}{4}\pi r^3$                       (d)  $\pi r^3$
- 13 Area of square having perimeter 40 cm is  $\underline{\hspace{2cm}}$   
 (a)  $1600 \text{ cm}^2$                       (b)  $20 \text{ cm}^2$                       (c)  $10 \text{ cm}^2$                       (d)  $100 \text{ cm}^2$
- 14 Area of circle having radius 4 cm is  $\underline{\hspace{2cm}}$   
 (a)  $4\pi$                       (b)  $16\pi$                       (c)  $8\pi$                       (d)  $2\pi$

Q:2 (A) Attempt any two 06

- (1) Prove that  $\log\left(\frac{x^a}{x^b}\right) + \log\left(\frac{x^b}{x^c}\right) + \log\left(\frac{x^c}{x^a}\right) = 0$
- (2) If the surface area of sphere is  $2826 \text{ cm}^2$  then find its radius (take  $\pi = 3.14$ )
- (3) Find the curved surface area of a cylinder having radius 3.5 cm and height 21 cm. (take  $\pi = 22/7$ )

(B) Attempt any two 08

- (1) prove that  $\frac{1}{\log_{12} 60} + \frac{1}{\log_{15} 60} + \frac{1}{\log_{20} 60} = 2$
- (2) If  $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$  then prove that  $a=b$
- (3) Diameter of a cylindrical tank is 7m. If  $385 \text{ m}^3$  water is fill in the tank. What will be the height of water in tank? (take  $\pi = 22/7$ )

Q:3 (A) Attempt any two 06

- (1) If  $\begin{vmatrix} x & 2 & 3 \\ 5 & 0 & 7 \\ 3 & 1 & 2 \end{vmatrix} = 30$  then find x.

- (2) If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 1 & 3 \\ 1 & -1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 3 & 4 \end{bmatrix}$  then find  $A+B-C$

(3) If  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  then prove that  $A^2 - 2A - 3I = 0$

(B) Attempt any two

08

(1) If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$  then find  $A^{-1}$

(2) Solve the given equations using matrix method.  $3x - y = 1$ ,  $2x + y = 4$ .

(3) If  $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$  then find  $(AB)^T$ .

Q:4 (A) Attempt any two

06

(1) Prove that  $\frac{\sin(\frac{\pi}{2} + \theta)}{\cos(2\pi - \theta)} + \frac{\tan(\pi + \theta)}{\cot(\frac{\pi}{2} - \theta)} + \frac{\sec(\frac{3\pi}{2} + \theta)}{\csc(\pi - \theta)} = 3$

(2) Prove that  $\tan 35^\circ + \tan 10^\circ + \tan 35^\circ \tan 10^\circ = 1$

(3) Prove that  $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(B) Attempt any two

08

(1) Draw the graph of  $y = \cos x$  where  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

(2) Prove that  $\frac{\cos \theta + \cos 3\theta + \cos 5\theta}{\sin \theta + \sin 3\theta + \sin 5\theta} = \cot 3\theta$

(3) Prove that  $\frac{\sin 3\theta - \sin \theta}{\cos 3\theta + \cos \theta} = \tan \theta$

Q:5 (A) Attempt any two

06

(1) If  $\vec{a} = (1, 2, 1)$ ,  $\vec{b} = (2, 1, 1)$  and  $\vec{c} = (3, 4, 1)$  then find  $|\vec{a} - 2\vec{b} + \vec{c}|$

(2) If  $\vec{a} = (p, 2, 1)$  and  $\vec{b} = (2, p, -4)$  are perpendicular to each other then find p.

(3) If  $\vec{x} + \vec{y} = (3, 1, 2)$  and  $\vec{x} - \vec{y} = (1, 1, 4)$  then find  $\vec{x} \cdot \vec{y}$

(B) Attempt any two

08

(1) Find unit perpendicular vector to  $\vec{a} = (2, -3, 4)$  and  $\vec{b} = (1, -1, 1)$

(2) Under the effect of two forces  $(3, 2, 1)$  and  $(1, 5, 2)$  a particle moves from  $(1, 3, -2)$  to  $(3, 1, 4)$ . Find the work done.

(3) If the angle between two vectors  $3\vec{i} + \vec{j} + 2\vec{k}$  and  $2\vec{i} - 2\vec{j} + 4\vec{k}$  is  $\theta$ . Prove  $\theta =$

$$\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$$

- 1  $\log_2 8 = \underline{\hspace{2cm}}$   
 (a) 3 (b) 0 (c) 1 (d) 2
- 2  $4^{\log_4 2} = \underline{\hspace{2cm}}$   
 (a) 0 (b) 4 (c) 2 (d) 1
- 3  $\log_3 2 \times \log_2 3 = \underline{\hspace{2cm}}$   
 (a) 3 (b) 1 (c)  $\log_2 5$  (d)  $\log_3 5$
- 4 જો  $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$  તો  $x = \underline{\hspace{2cm}}$   
 (a) -2 (b) 2 (c) -4 (d) 4
- 5  $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$  નો ક્રમ  $\underline{\hspace{2cm}}$  છે.  
 (a)  $2 \times 3$  (b)  $2 \times 2$  (c)  $3 \times 2$  (d)  $3 \times 3$
- 6 જો  $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$  તો  $A^T = \underline{\hspace{2cm}}$   
 (a)  $A^{-1}$  (b)  $A$  (c)  $\begin{bmatrix} 2 & 4 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 4 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$
- 7  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \underline{\hspace{2cm}}$   
 (a)  $\begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$  (b)  $\begin{bmatrix} 5 & 3 \\ 9 & 7 \end{bmatrix}$  (c)  $\begin{bmatrix} 3 & 7 \\ 5 & 9 \end{bmatrix}$  (d) એક પણ નહિ.
- 8  $\sin\left(\frac{\pi}{2} - \theta\right) = \underline{\hspace{2cm}}$   
 (a)  $-\cos\theta$  (b)  $\cos\theta$  (c)  $\sin\theta$  (d)  $-\sin\theta$
- 9  $\sin^{-1} x + \cos^{-1} x = \underline{\hspace{2cm}}$   
 (a) 0 (b) 1 (c)  $\pi$  (d)  $\frac{\pi}{2}$
- 10  $\sin 2x$  નું આવર્તમાન  $\underline{\hspace{2cm}}$  છે.  
 (a)  $2\pi$  (b)  $\frac{\pi}{2}$  (c)  $\pi$  (d) 2
- 11  $\sin 2\theta = \underline{\hspace{2cm}}$   
 (a)  $2\sin\theta$  (b)  $2\sin\theta\cos\theta$  (c)  $\sin^2\theta - \cos^2\theta$  (d)  $\cos^2\theta - \sin^2\theta$
- 12 'r' ત્રિજ્યા વાળા ગોલક નું ઘનફળ  $\underline{\hspace{2cm}}$  થાય.

(a)  $\frac{4}{3}\pi r^3$       (b)  $\frac{1}{3}\pi r^3$       (c)  $\frac{3}{4}\pi r^3$       (d)  $\pi r^3$

13 40 cm પરિમિતિ વાળા ચોરસ નું ક્ષેત્રફળ \_\_\_\_\_ થાય.

(a) 1600 cm<sup>2</sup>      (b) 20 cm<sup>2</sup>      (c) 10 cm<sup>2</sup>      (d) 100 cm<sup>2</sup>

14 4 cm ત્રિજ્યા વાળા વર્તુળનું ક્ષેત્રફળ \_\_\_\_\_ થાય.

(a) 4π      (b) 16π      (c) 8π      (d) 2π

Q: (A) ગમે તે બે ગણો. 0

2 ) 6

(1) સાબિત કરો કે  $\log\left(\frac{x^a}{x^b}\right) + \log\left(\frac{x^b}{x^c}\right) + \log\left(\frac{x^c}{x^a}\right) = 0$

(2) જો ગોલકનું વક્રસપાટી નું ક્ષેત્રફળ 2826cm<sup>2</sup> તો તેની ત્રિજ્યા શોધો(π=3.14)

(3) 3.5cm ત્રિજ્યા અને 21cm ઊંચાઈ વાળા નળાકારની વક્રસપાટી નું ક્ષેત્રફળ શોધો (π=22/7 લો)

(B) ગમે તે બે ગણો. 0

) 8

(1) સાબિત કરો કે  $\frac{1}{\log_{12} 60} + \frac{1}{\log_{15} 60} + \frac{1}{\log_{20} 60} = 2$

(2) જો  $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$  તો સાબિત કરો કે a=b

(3) નળાકાર ટાંકી નો વ્યાસ 7m છે. જો 385 m<sup>3</sup> પાણી આ ટાંકીમાં ભરવામાં આવે તો ટાંકી માં પાણીની ઊંચાઈ કેટલી થશે? (π=22/7 લો)

Q: (A) ગમે તે બે ગણો. 0

3 ) 6

(1) જો  $\begin{vmatrix} x & 2 & 3 \\ 5 & 0 & 7 \\ 3 & 1 & 2 \end{vmatrix} = 30$  તો x શોધો.

(2) જો  $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 2 & 1 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 1 & 3 \\ 1 & -1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ ,  $C = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 3 & 4 \end{bmatrix}$  તો A+B-C શોધો

(3) જો  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$  તો સાબિત કરો કે  $A^2 - 2A - 3I = 0$

(B ગમે તે બે ગણો. 0  
) 8

(1) જો  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$  તો  $A^{-1}$  શોધો.

(2) શ્રેણિકની મદદથી  $3x-y=1$ ,  $2x+y=4$  નો ઉકેલ શોધો.

(3) જો  $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$  તો  $(AB)^T$  શોધો.

Q: (A ગમે તે બે ગણો. 0  
4 ) 6

(1) સાબિત કરો કે  $\frac{\sin(\frac{\pi}{2} + \theta)}{\cos(2\pi - \theta)} + \frac{\tan(\pi + \theta)}{\cot(\frac{\pi}{2} - \theta)} + \frac{\sec(\frac{3\pi}{2} + \theta)}{\operatorname{cosec}(\pi - \theta)} = 3$

(2) સાબિત કરો કે  $\tan 35^\circ + \tan 10^\circ + \tan 35^\circ \tan 10^\circ = 1$

(3) ) સાબિત કરો કે  $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$

(B ગમે તે બે ગણો. 0  
) 8

(1)  $y = \cos x$  નો આલેખ દોરો. જ્યાં  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

(2) સાબિત કરો કે  $\frac{\cos \theta + \cos 3\theta + \cos 5\theta}{\sin \theta + \sin 3\theta + \sin 5\theta} = \cot 3\theta$

(3) સાબિત કરો કે  $\frac{\sin 3\theta - \sin \theta}{\cos 3\theta + \cos \theta} = \tan \theta$

Q: (A ગમે તે બે ગણો. 0  
5 ) 6

(1) જો  $\vec{a} = (1, 2, 1)$ ,  $\vec{b} = (2, 1, 1)$  અને  $\vec{c} = (3, 4, 1)$  તો  $|\vec{a} - 2\vec{b} + \vec{c}|$  શોધો.

(2) જો  $\vec{a} = (p, 2, 1)$  અને  $\vec{b} = (2, p, -4)$  એક બીજાને લંબ હોય તો  $p$  શોધો.

(3) જો  $\vec{x} + \vec{y} = (3, 1, 2)$  અને  $\vec{x} - \vec{y} = (1, 1, 4)$  તો  $\vec{x} \cdot \vec{y}$  શોધો.

(B ગમે તે બે ગણો. 0

- (1)  $\vec{a} = (2, -3, 4)$  અને  $\vec{b} = (1, -1, 1)$  ને લંબ એકમ સદિશ શોધો.
- (2) બંને  $(3, 2, 1)$  અને  $(1, 5, 2)$  ની અસર થી એક કણ બિન્દુ  $(1, 3, -2)$  થી  $(3, 1, 4)$  સુધી ગતિ કરે તો થયેલ કાર્ય શોધો.
- (3) સદિશ  $3\vec{i} + \vec{j} + 2\vec{k}$  અને  $2\vec{i} - 2\vec{j} + 4\vec{k}$  વચ્ચે નો ખૂણો  $\theta$  હોય તો સાબિત કરો કે

$$\theta = \sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$$