

GUJARAT TECHNOLOGICAL UNIVERSITY**Diploma Engineering - SEMESTER-I • EXAMINATION – WINTER 2013****Subject Code: 3300001****Date: 17-12-2013****Subject Name: Basic Mathematics****Time: 02:30 pm - 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options. **14**

- 1 $\log_b a \times \log_a b = \dots\dots\dots$
(i) 0 (ii) 1 (iii) $\log_a ab$ (iv) $\log_b ab$
- 2 If $\log_2 x = 5$ then $x = \dots\dots\dots$
(i) 32 (ii) 25 (iii) $\frac{2}{5}$ (iv) $\frac{5}{2}$
- 3 $1024^{\log_2 m} \dots\dots\dots$
(i) 10^m (ii) m^{10} (iii) 2^{10} (iv) 10^2
- 4 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\text{adj } A = \dots\dots\dots$
(i) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (ii) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (iii) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (iv) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$
- 5 If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^T = \dots\dots\dots$
(i) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$
- 6 If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ then cofactor of 5 = $\dots\dots\dots$
(i) -12 (ii) 12 (iii) -3 (iv) 3
- 7 If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^2 = \dots\dots\dots$
(i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
- 8 $\sin 120^\circ = \dots\dots\dots$
(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{1}{2}$ (iii) $\frac{\sqrt{3}}{2}$ (iv) $\sqrt{3}$
- 9 $\tan^{-1}(\sqrt{3}) = \dots\dots\dots$
(i) 30° (ii) 45° (iii) 60° (iv) 120°
- 10 Period of $\sin(2x + 3) = \dots\dots\dots$
(i) $\frac{2\pi}{3}$ (ii) 2π (iii) π (iv) $\frac{\pi}{2}$

- 11 $\sin(A + B) \sin(A - B) = \dots\dots\dots$
 (i) $\sin^2 A - \cos^2 B$ (ii) $\sin^2 A - \sin^2 B$ (iii) $\cos^2 A - \cos^2 B$
 (iv) $\cos^2 A - \sin^2 B$
- 12 Area of a square is 100 sq-cm. Perimeter of square = $\dots\dots\dots$
 (i) 10cm (ii) 20cm (iii) 40cm (iv) 60cm
- 13 Circumference of a circle is 10π cm. Radius of a circle = $\dots\dots\dots$
 (i) 5cm (ii) 25cm (iii) 10cm (iv) 100cm
- 14 Volume of sphere having radius $r = \dots\dots\dots$
 (i) $\frac{4}{3}\pi r^2$ (ii) $\frac{4}{3}\pi r^3$ (iii) $\frac{3}{4}\pi r^3$ (iv) $4\pi r^2$

Q.2 (a) Attempt any two **06**

- If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that $a = b$
- If the circumference of a circle is equal to the area of a circle, find the radius of a circle.
- The surface area of a sphere is 616 sq-cm, find the diameter of the sphere.

(b) Attempt any two **08**

- Prove that $\frac{1}{\log_x(yz)+1} + \frac{1}{\log_y(zx)+1} + \frac{1}{\log_z(xy)+1} = 1$
- Solve: $\log x + \log(x - 5) = \log 6$
- If the surface area of a spherical ball is 1256 sq-cm, find the volume of the sphere

Q.3 (a) Attempt any two **06**

- If $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$, then find $(AB)^T$
- If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$, then find $3A - 2B$
- Evaluate: $\begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

(b) Attempt any two **08**

- If $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$, $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$, then find $(AB)^{-1}$
- Solve: $2x - 3y = -5$, $3x + 5y = 9$ using matrices
- If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ then find A^2

Q.4 (a) Attempt any two **06**

- Prove that $\frac{\sin(\pi+\theta)}{\sin(2\pi-\theta)} + \frac{\tan(\frac{\pi}{2}+\theta)}{\cot(\pi-\theta)} + \frac{\cos(2\pi+\theta)}{\sin[\frac{\pi}{2}+\theta]} = 3$
- Prove that $\tan 20^\circ + \tan 25^\circ + \tan 20^\circ \tan 25^\circ = 1$
- Prove that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(b) Attempt any two **08**

- Draw the graph of $y = \cos x$, $0 \leq x \leq \pi$
- If $\tan \theta = \frac{1}{2}$ then find $7 \cos 2\theta + 8 \sin 2\theta$
- Prove that $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$

- Q.5 (a)** Attempt any two **06**
- If $a = (3 \ -1 \ -4)$, $b = (-2 \ 4 \ -3)$, $c = (1 \ 2 \ 1)$
Find $|3a - 2b + 4c|$
 - If $x = (1 \ -2 \ 3)$, $y = (-2 \ 3 \ 1)$ then find $(x + y) \cdot (x - y)$
 - Find x if $a = (2 \ -3 \ 5)$ and $b = (x \ -6 \ -8)$ are perpendicular
To each other
- (b)** Attempt any two **08**
- Find a unit vector perpendicular to both $a = (1 \ -1 \ 1)$ and
 $b = (2 \ 3 \ -1)$
 - Prove that the angle subtended between $(1 \ 1 \ -1)$ and $(2 \ -2 \ 1)$ is
 $\sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$
 - The constant forces $(1 \ 2 \ 3)$ and $(3 \ 1 \ 1)$ act on a particle. Under the
action of these forces particle moves to the point $(5 \ 1 \ 2)$ from the point
 $(0 \ 1 \ -2)$. Find the total work done

- Q.1** સાચો વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો. **14**
- $\log_b a \times \log_a b = \dots\dots\dots$
(i) 0 (ii) 1 (iii) $\log_a ab$ (iv) $\log_b ab$
 - જો $\log_2 x = 5$ તો $x = \dots\dots\dots$
(i) 32 (ii) 25 (iii) $\frac{2}{5}$ (iv) $\frac{5}{2}$
 - $1024^{\log_2 m} \dots\dots\dots$
(i) 10^m (ii) m^{10} (iii) 2^{10} (iv) 10^2
 - જો $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ તો $\text{adj } A = \dots\dots\dots$
(i) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (ii) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (iii) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (iv) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$
 - જો $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ તો $A^T = \dots\dots\dots$
(i) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$
 - જો $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ તો 5 નો સહઅવયવ = $\dots\dots\dots$
(i) -12 (ii) 12 (iii) -3 (iv) 3
 - જો $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ તો $A^2 = \dots\dots\dots$
(i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
 - $\sin 120^\circ = \dots\dots\dots$
(i) $\frac{2}{\sqrt{3}}$ (ii) $\frac{1}{2}$ (iii) $\frac{\sqrt{3}}{2}$ (iv) $\sqrt{3}$
 - $\tan^{-1}(\sqrt{3}) = \dots\dots\dots$
(i) 30° (ii) 45° (iii) 60° (iv) 120°

- 10 $\sin(2x + 3)$ નું અવર્તમાન =
- (i) $\frac{2\pi}{3}$ (ii) 2π (iii) π (iv) $\frac{\pi}{2}$
- 11 $\sin(A + B)\sin(A - B) = \dots\dots\dots$
- (i) $\sin^2 A - \cos^2 B$ (ii) $\sin^2 A - \sin^2 B$ (iii) $\cos^2 A - \cos^2 B$
(iv) $\cos^2 A - \sin^2 B$
- 12 ચોરસનું ક્ષેત્રફળ 100 ચો.સેમી છે. ચોરસની પરિમિત =
- (i) 10સેમી (ii) 20સેમી (iii) 40સેમી (iv) 80સેમી
- 13 વર્તુળનો પરિઘ 10π સેમી છે. વર્તુળની ત્રિજ્યા =
- (i) 5સેમી (ii) 25સેમી (iii) 10સેમી (iv) 100સેમી
- 14 r ત્રિજ્યાવાળા ગોળકનું ઘનફળ =
- (i) $\frac{4}{3}\pi r^2$ (ii) $\frac{4}{3}\pi r^3$ (iii) $\frac{3}{4}\pi r^3$ (iv) $4\pi r^2$

Q.2 (a) કોઈપણ બે ગણો. **06**

1. જો $\log\left(\frac{a+b}{2}\right) = \frac{1}{2}(\log a + \log b)$ તો સાબિત કરો કે $a = b$
2. વર્તુળનો પરિઘ વર્તુળનાં ક્ષેત્રફળ જેટલો છે. વર્તુળની ત્રિજ્યા શોધો.
3. ગોળકની સપાટીનું ક્ષેત્રફળ 616 ચો.સેમી છે. ગોળકની મેલવો.

(b) કોઈપણ બે ગણો. **08**

1. સાબિત કરો કે $\frac{1}{\log_x(yz)+1} + \frac{1}{\log_y(zx)+1} + \frac{1}{\log_z(xy)+1} = 1$
2. ઉકેલો ; $\log x + \log(x - 5) = \log 6$
3. ગોલાકાર ટાંચની સપાટીનું ક્ષેત્રફળ 1256 ચો.સેમી છે. ગોલાકાર ટાંચનું ઘનફળ મેલવો.

Q.3 (a) કોઈપણ બે ગણો. **06**

1. જો $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$ તો $(AB)^T$ મેલવો.

2. જો If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ તો $3A-2B$ મેલવો.

3. મેલવો : $[2 \ 1 \ 1] \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

(b) કોઈપણ બે ગણો **08**

1. જો $A+B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$, $A-B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$ તો $(AB)^{-1}$ મેલવો.
2. શ્રેણિકની મદદથી ઉકેલો ; $2x-3y = -5$, $3x + 5y = 9$
3. જો $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ તો A^2 મેલવો.

Q.4 (a) કોઈપણ બે ગણો **06**

1. સાબિત કરો કે $\frac{\sin(\pi+\theta)}{\sin(2\pi-\theta)} + \frac{\tan(\frac{\pi}{2}+\theta)}{\cot(\pi-\theta)} + \frac{\cos(2\pi+\theta)}{\sin[\frac{\pi}{2}+\theta]} = 3$

2. સાબિત કરો કે $\tan 20^\circ + \tan 25^\circ + \tan 20^\circ \tan 25^\circ = 1$

3. સાબિત કરો કે $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(b) કોઈપણ બે ગણો **08**

1. $y = \cos x$, $0 \leq x \leq \pi$ નો ગ્રાફ દોરો

2. જો $\tan \theta = \frac{1}{2}$ તો $7 \cos 2\theta + 8 \sin 2\theta$ મેળવો

3. સાબિત કરો કે $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$

Q.5 (a) કોઈપણ બે ગણો **06**

1. જો $a = (3 \ -1 \ -4)$, $b = (-2 \ 4 \ -3)$, $c = (1 \ 2 \ 1)$ તો $|3a - 2b + 4c|$ મેળવો

2. જો $x = (1 \ -2 \ 3)$, $y = (-2 \ 3 \ 1)$ તો $(x+y) \cdot (x-y)$ મેળવો

3. જો $a = (2 \ -3 \ 5)$ and $b = (x \ -6 \ -8)$ પરસ્પર લંબ હોય તો x મેળવો

(b) કોઈપણ બે ગણો **08**

1. $a = (1 \ -1 \ 1)$ અને $b = (2 \ 3 \ -1)$ ને લંબ આવેલો એકમ સદિશ મેળવો

2. સાબિત કરો કે સદિશો $(1 \ 1 \ -1)$ અને $(2 \ -2 \ 1)$ અંતરાયેલો ખુણો

$\sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$ છે.

3. એક કણ ઉપર અચળ બળો $(1 \ 2 \ 3)$ અને $(3 \ 1 \ 1)$ કાર્ય કરે છે. આ બળોની અસર હેથળ તે કણ બિંદુ $(0 \ 1 \ -2)$ થી ખસીને બીજા બિંદુ $(5 \ 1 \ 2)$ આગળ આવે છે. કુલ કાર્ય શોધો.

c