

GUJARAT TECHNOLOGICAL UNIVERSITY
MCA - SEMESTER-III • EXAMINATION – SUMMER - 2017

Subject Code: 3630001**Date: 02/06/2017****Subject Name: Basic Mathematics****Time: 02:30 pm - 05:00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Show the following equivalences without constructing the truth table **04**
- (i) $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ **04**
- (ii) $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$ **03**
- (b) (i) State Peano's axioms. **02**
- (ii) Prove that $1+2+3+\dots+n = n(n+1)/2$ **03**
- (iii) Show that $n^3 + 2n$ is divisible by 3 **02**
- Q.2** (a) Define: Maximal Compatibility Block. Let the compatibility relations on the sets $\{1, 2, 3, 4, 5, 6\}$ and $\{x_1, x_2, \dots, x_6\}$ be given by following two matrices respectively. Construct the graph and find the maximum compatibility blocks. **07**

(i)	$\begin{array}{c c} 2 & 1 \\ 3 & 1 \ 1 \\ 4 & 1 \ 1 \ 1 \\ 5 & 0 \ 1 \ 0 \ 0 \\ 6 & 0 \ 0 \ 1 \ 0 \ 1 \\ \hline & 1 \ 2 \ 3 \ 4 \ 5 \end{array}$	(ii)	$\begin{array}{c c} x_2 & 1 \\ x_3 & 1 \ 1 \\ x_4 & 0 \ 0 \ 1 \\ x_5 & 0 \ 0 \ 1 \ 1 \\ x_6 & 1 \ 0 \ 1 \ 0 \ 1 \\ \hline & x_1 \ x_2 \ x_3 \ x_4 \ x_5 \end{array}$
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- (b) Construct the truth tables for the following formulas **04**
- (i) $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$ **04**
- (ii) $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ **03**
- OR**
- (b) Define Relation. Let $X = \{1, 2, 3, 4\}$ and $R = \{\langle x, y \rangle \mid x > y\}$. Draw the graph of R and also give its matrix. **07**
- Q.3** (a) Draw the Hasse diagrams of the following sets under the partial ordering relation "divides" and indicate those which are totally ordered. **07**
- (i) $\{1, 2, 3, 4\}$ (ii) $\{3, 5, 15\}$ (iii) $\{2, 4, 8, 16\}$
- (iv) $\{1, 2, 3, 6, 12\}$ (v) $\{2, 3, 6, 12, 24, 36\}$
- (b) (i) For $A = \{2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{4, 5, 6, 7, 8\}$ find **04**
- a) $(A \cup B) \cap (A \cup C)$ b) $(A \cap B) \cup (A \cap C)$
- (ii) Define Power Set. Find the Power Set of the set $Q = \{1, \{2, 3\}, 4\}$ **03**

OR

- Q.3** (a) Define Composition of a function. Let $X = \{1, 2, 3\}$ and f, g, h and s be functions from X to X given by **07**
- $f = \{\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle\}$ $g = \{\langle 1, 2 \rangle, \langle 2, 1 \rangle, \langle 3, 3 \rangle\}$
- $h = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 1 \rangle\}$ $s = \{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\}$
- Find $f \circ g$, $g \circ f$, $f \circ h \circ g$, $s \circ g$, $g \circ s$, $s \circ s$
- (b) What is Recursive Function? Write a Recursive algorithm to find out Fibonacci series. **07**

- Q.4 (a)** Define equivalence relation. Let $X = \{1, 2, \dots, 7\}$ and $R = \{\langle x, y \rangle \mid x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation. Draw the graph of R . **07**
- (b)** Find the inverse of the matrix **07**

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

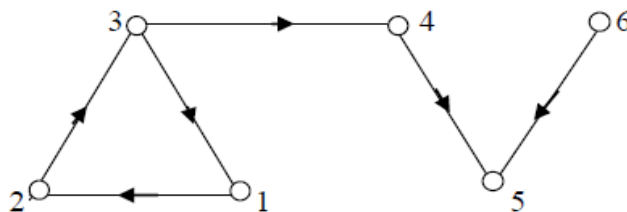
OR

- Q.4 (a)** Explain with example injective (onto), surjective (one-to-one) and bijective (one-to-one onto) function. Let N be set of Natural numbers including zero. Determine whether the function given below is injective, surjective or bijective. **07**

$$f: N \rightarrow N \quad f(j) = j^2 + 2$$

- (b)** (i) If $A = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$ Show that $A^3 = I$ and so find A^{-1} **04**
- (ii) Show that $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ is the inverse of $\begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$ **03**

- Q.5 (a)** (i) Define Cyclic graph, Null graph, and Strongly connected graph. **03**
- (ii) Define Adjacency matrix and path matrix of a graph. Explain each with example. **04**
- (b)** Define a unilateral component and strong component. Write unilateral and strong and weak components of the Graph given in following figure. **07**



OR

- Q.5 (a)** Define a directed tree. Draw the graph of the tree represented by $(A(B(E(H)(I))(F(J)(K))(G(L)))(C(M(O))(N(P)(Q)))(D(R(S(V))(T)(U))))$ Obtain the binary tree corresponding to it. **07**
- (b)** (i) Define a path in graph. Define length of the path. What is difference between a simple path and an elementary path? **03**
- (ii) Define isomorphic graphs. State whether the following graphs are isomorphic or not. **04**

