Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE SEMESTER 1st / 2nd (NEW) EXAMINATION WINTER 2016

Subject Code: 2110014   Date: 24/01/2017
Subject Name: Calculus
Time:10:30 AM TO 1:30 PM   Total Marks: 70

Instructions:
1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1   **Objective Question (MCQ)**   Mark

(a)   **07**

1. The sequence $sin\left(\frac{\pi}{6} + \frac{1}{n}\right)$ converges to
    (a) 0   (b) 1   (c) -1   (d) 0.5

2. The sum of the series $\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$ is ---
    (a) $\frac{\pi}{\pi-e}$ (b) $\frac{e}{\pi-e}$ (c) $\frac{\pi}{e-\pi}$ (d) $\frac{e}{\pi}$

3. The value of the limit $\lim_{x\to 6}\frac{sin(x-6)}{x-6}$ = ---
    (a) 0   (b) 1   (c) -1   (d) 0.5

4. Asymptote parallel to Y-axis of the curve $y = \frac{x^2}{x-3}$ is the line ---
    (a) x = 0   (b) y = 3   (c) x = 3   (d) not exist

5. Let $f(x, y) = y\, sin(xy)$. The value of $f_x(\pi, 1)$ is ---
    (a) 0   (b) 1   (c) -1   (d) 2.5

6. $\int_1^2\int_1^2 \frac{1}{xy}\,dxdy$ = ---
    (a) 0   (b) $(log2)^2$ (c) 1   (d) $log2$

7. The coefficient of x^5 in the expansion of e^x is ---
    (a) $\frac{1}{5}$ (b) $\frac{1}{4!}$ (c) $\frac{1}{5!}$ (d) 5

(b)   **07**

1. $\sum_{n=1}^{\infty}\frac{2n}{3n-1}$ is ---
    (a) convergent and sum is 0   (b) convergent and sum is 1
    (c) divergent   (d) oscillating

2. If $f = e^{x/y}tan\left(\frac{x}{y}\right) + e^{y/x}cot\left(\frac{y}{x}\right)$, then value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is ---
    (a) 0   (b) f (c) $-f$ (d) $2f$

3. The curve $x^3y + y^3x = 3$ is symmetric about the ---
    (a) X-axis   (b) Y-axis   (c) origin   (d) line y = x

4. What does the region of $\int_1^4\int_2^6 dxdy$ represents?
    (a) rectangle   (b) square   (c) circle   (d) triangle

5. The value of $\int_0^{\infty}\frac{1}{x^2+1}dx$ = ---
    (a) π (b) $\frac{\pi}{2}$ (c) 0   (d) 1

6. The minimum value of $f(x, y) = x^2 + y^2$ is ---
    (a) 1   (b) 2   (c) 4   (d) 0

7. If $x = u + 3v$, $y = v - u$ then value of $J = \frac{\partial(x,y)}{\partial(u,v)}$ is ---
    (a) -1   (b) 4   (c) 5   (d) 7

Q.2 **(a)** You drop a ball from a meters above a flat surface. Each time the ball hits the surface after falling a distance h, it rebounds a distance rh, where $0 < r < 1$. Find the total distance ball travels up and down when $a = 6$ m and $r = 2/3$ m. **03**

(b) Evaluate **04**

(1) $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x} - x^2 - 2}{sin^2 x - x^2}$ (2) $\lim\limits_{x \to 1}\left(\dfrac{x}{x-1} - \dfrac{1}{logx}\right)$

(c) Obtain the Maclaurin's series of $log_e(1+x)$ and hence find the series of $log_e\left(\dfrac{1+x}{1-x}\right)$ and then obtain approximate value of $log_e\left(\dfrac{11}{9}\right)$. **07**

Q.3 **(a)** Show that $f(x,y) = \begin{cases} \dfrac{2x^2 y}{x^3 + y^3}, & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}$ is not continuous at the origin. **03**

(b) If $\theta = t^n e^{-r^2/4t}$, then find the value of n for which $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial \theta}{\partial r}\right) = \dfrac{\partial \theta}{\partial t}$. **.04**

(c) State Euler's theorem on homogenous function of two variables. If $u = tan^{-1}\left(\dfrac{x^2 + y^2}{x+y}\right)$, prove that **07**

$x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = -2sin^3 u cos u.$

Q.4 **(a)** Find the Jacobian of the transformation **03**
$x = \rho sin\varphi cos\theta,\ y = \rho sin\varphi sin\theta,\ z = \rho cos\varphi.$

(b) Find the tangent plane and normal line of the surface $f(x,y,z) = x^2 + y^2 + z - 9 = 0$ at the point P(1, 2, 4). **04**

(c) A rectangular box open at the top is to have a volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction. **07**

Q.5 **(a)** Evaluate $\int_{-1}^{1}\int_{0}^{2}(1 - 6x^2 y)dxdy$. **03**

(b) Evaluate $\int_{0}^{1}\int_{0}^{\sqrt{z}}\int_{0}^{2\pi}(r^2 cos^2\theta + z^2)rd\theta drdz$. **04**

(c) Change the order of integration and evaluate **07**

$$\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1)\sqrt{1 - x^2 - y^2}}dydx$$

Q.6 **(a)** Let $S = \sum_{n=1}^{\infty} n\alpha^n$ where $|\alpha| < 1$. Find the value of α in (0, 1) such that $S = 2\alpha$. **03**

(b) Check for convergence/divergence **04**

(1) $\sum_{n=1}^{\infty}\dfrac{5n^3 - 3n}{n^2(n-2)(n^2+5)}$ (2) $\sum_{n=1}^{\infty}\dfrac{2^n}{n^3+1}$

(c) (1) Check for absolute/conditional convergence of $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$. **03**

(2) For the series $\sum_{n=1}^{\infty}(-1)^{n-1}\dfrac{x^{2n-1}}{2n-1}$, find the radius and interval of convergence. **04**

Q.7 **(a)** The graph of $y = x^2$ between x = 1 and x = 2 is rotated around the X-axis. Find the volume of a solid so generated.   03

(b) Test the convergence of the improper integrals. If convergent then evaluate the same.   04

(1) $\int_0^1 \frac{dx}{1-x}$ (2) $\int_0^\infty \frac{1}{(1+x^2)(1+ tan^{-1}x)} dx$

(c) Trace the curve $y^2(a+x) = x^2(a-x)$, $a > 0$.   07

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