Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE SEMESTER– 1st/2nd (NEW SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code:2110014   **Date: 02/06/2015**

Subject Name: Calculus

Time:10.30am-01.00pm   **Total Marks: 70**

Instructions:
1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1   **Objective Question (MCQ)**✱✱(see below for note on Q.1)

     (a)   **Attempt the following.**   **07**

     1.   Maclaurin series of $\sin x$ is

a) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$ b) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n+1}}{(2n+1)!}$ c) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$ d) $\displaystyle\sum_{n=0}^{\infty}(-1)^{n}\frac{x^{2n}}{(2n)!}$

     2.   The total area enclosed between $y=\sin x$ and $X-$axis in $[0,\pi]$ is

a)   0   b)   4   c)   2   d)   1

     3.   The value of $\displaystyle\lim_{x\to\infty}x^{1/x}$

a) ∞ b) $-\infty$ c)   1   d)   0

     4.   How many leaves $r=\sin 5\theta$ has ?

a)   6   b)   2   c)   3   d)   5

     5.   The sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^{n}}$ is

a)   0   b)   1   c)   -1   d) $\frac{1}{2}$

     6.   The values of x at which the curves $y=x$ & $y=4x-x^{2}$ intersects each other are

a)   0 & 4   b)   -4 & 4   c)   0 & 3   d)   -3 & 3

     7.   The area enclosed between the curve $r=f(\theta)$ and two rays $\theta=\alpha$ & $\theta=\beta$ is

a) $\displaystyle\int_{\alpha}^{\beta}r^{2}d\theta$ b) $\displaystyle\int_{\alpha}^{\beta}r^{3}d\theta$ c) $\displaystyle\frac{4\pi}{3}\int_{\alpha}^{\beta}r^{3}d\theta$ d) $\displaystyle\frac{1}{2}\int_{\alpha}^{\beta}r^{2}d\theta$

     (b)   **Attempt the following.**   **07**

     1.   If $u=\sin^{-1}\dfrac{x}{y}+\tan^{-1}\dfrac{y}{x}$ then the value of $xu_{x}+yu_{y}$ is

a) u b) $-u$ c)   0   d)   1

     2.   For an implicit function $f(x,y)=c$ the value of $\dfrac{dy}{dx}$ is

a) $\dfrac{f_{x}}{f_{y}}$ b) $\dfrac{f_{y}}{f_{x}}$ c) $-\dfrac{f_{x}}{f_{y}}$ d) $-\dfrac{f_{y}}{f_{x}}$

     3.   For the function $u=\left(x^{2}+y^{2}\right)^{1/3}$ the value of $x^{2}u_{xx}+2xyu_{xy}+y^{2}u_{yy}$ is

a) $-2u$ b) $-\dfrac{2u}{3}$ c) $\dfrac{u}{9}$ d) $-\dfrac{2u}{9}$

     4.   If $x=r\cos\theta$ & $y=r\sin\theta$ then $J=\dfrac{\partial(r,\theta)}{\partial(x,y)}$ is

a) r b) $-r$ c) $\dfrac{1}{r}$ d) $-\dfrac{1}{r}$

     5.   The fundamental period of $\sin 2x$ is

a) π b) 2π c) 4π d) $\dfrac{\pi}{2}$

6. The focus of parabola $y^2 4ax$ is $(ae,0)$ if

  a) $e<1$ b) $e>1$ c) $e=1$ d) $e\neq1$

7. The function $f(x)=2x^3+3x^2-12x+7$ is decreasing in

  a) $[-2,1]$ b) $R-[-2,1]$ c) $[0,2]$ d) $[1,3]$

OR

Q.1 Objective Question (MCQ)**(**14**

1 $\displaystyle\lim_{x\to2}\dfrac{x^2-4}{x^2+4}=$

  (a) 1   (b) 0   (c) -1/2   (d) None of these

2 $f(x)=\dfrac{x^2-x}{2x};x\neq0$

$f(0)=k$

If

and if f is continuous on $x=0$ then $k=$

  (a) -1   (b) -1/2   (c) 0   (d) None of these

3 What is the slope of the tangent line to the curve $x+y=xy$ at point $(2,2)$

  (a) -1   (b) -2   (c) -3   (d) -4

4 Determine the second derivative of the function $f(x)=x^2\cdot\ln 2x$

  (a) $2\ln 2x+3$ (b) $2\ln 2x+\dfrac{3}{2}$ (c) 0   (d) None of these

5 At a minimum, the second differential function of the form $y=ax^n+bx^{n-1}+.....$ is

  (a) Positive   (b) Negative   (c) Zero   (d) Infinite

6 If $y=\dfrac{3}{x^4}$ then $\int y\,dx$

  (a) $\dfrac{15}{x^5}+c$ (b) $-\dfrac{1}{x^3}+c$ (c) $\dfrac{12}{x^3}+c$ (d) None of these

7 Evaluate $y=\displaystyle\int_0^{\frac{\pi}{2}}\sin(4x)\sin(6x)\,dx$

  (a) $y=\dfrac{\pi}{2}$ (b) $y=\dfrac{\pi}{4}$ (c) $y=\dfrac{3\pi}{4}$ (d) $y=0$

8 Use matrices to solve the following simultaneous equations

$x+2z=9$

$4x+2y+z=14$

$x+3y+4z=26$

  (a) $x=3;y=-1/2;z=3$ (b) $x=-1;y=1;z=6$

  (c) $x=1;y=3;z=4$ (d) $x=1;y=4;z=2$

9 Determine the area bounded by

The x axis, the curve $y = \sin 2x$,and the line $x = \dfrac{\pi}{4}$ and $x = \pi/2$
    (a) 0.5   (b) 1   (c) 0.25   (d) 1.5

10 Use matrices to solve the following simultaneous equations

$x + 2y = 3$

$2x + 3y = 5$

    (a) $x = 2; y = 1$ (b) $x = 1; y = 1$

    (c) $x = 2; y = 2$ (d) None of these

11 Determine the area bounded by

The x axis, the curve $y = 2x^2 + x - 6$,and the line $x = 4$ and $x = 6$

    (a) 298   (b) 126   (c) $99\dfrac{1}{3}$ (d) $26\dfrac{2}{3}$

12 Given the function $f(x) = x^2$ which value of c satisfies the conclusion of mean value theorem on the interval [-4, 5]?

    (a) 0   (b) 1   (c) 1/2   (d) None of these

13 Eliminate the parameter in the equations $x = t^2; y = t^4$

    (a) $y = x^2 \ for \ x \geq 0$ (b) $y = \sqrt{x} for \ x \geq 0$ (c) $y = 2x^2 \ for \ x \geq 0$ (d) None of these

14 At how many places does the curve $x = \cos 3t; y = \sin t$ cross the x-axis?

    (a) 2   (b) 1   (c) 3   (d) None of these

Q.2 (a) Show that the $p - series$ $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ ($p -$ a real constant) converges if $p > 1$ and diverges if $p \leq 1$.   **03**

(b) Define the Geometric series and find the sum of the following series $\displaystyle\sum_{n=1}^{\infty} \dfrac{3^{n-1} - 1}{6^{n-1}}$.   **04**

(c) Attempt the following.

  1) Investigate the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{2^n + 5}{3^n}$.   **03**

  2) Is the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{2n+1}{(n+1)^2}$ converges or diverges?   **04**

Q.3 (a) Define Taylor's series for the function of one variable and using it show that   **03**

$$\tan^{-1}(x+h) = \tan^{-1} x + (h\sin\alpha)\dfrac{\sin\alpha}{1} - (h\sin\alpha)^2 \dfrac{\sin 2\alpha}{2} + (h\sin\alpha)^3 \dfrac{\sin 3\alpha}{3} + ----.$$

Where $\alpha = \cot^{-1} x$

(b) Trace the curve $9ay^2 = x(x - 3a)^2$.   **04**

(c) Attempt the following.   **03**

  1) Evaluate $\displaystyle\lim_{x \to 0}\left[\dfrac{a^x + b^x + c^x}{3}\right]^{1/3x}$.   **03**

  2) Define volume of solid of revolution by washer's method and use it to find the volume of solid generated when the region between the graphs $f(x) = \dfrac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about $X -$ axis.   **04**

Q.4

(a) Define Improper integral of both the kinds. Check the convergence of $\displaystyle\int_0^3 \frac{dx}{\sqrt{9-x^2}}$. **03**

(b) Trace the curve $r^2 = a^2 \cos 2\theta$. **04**

(c) **Attempt the following.**

  1) Evaluate $\displaystyle\lim_{x \to 0}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$. **03**

  2) Define the volume of solid of revolution by disk and use it to find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1, 4]$ is revolved about $X-$axis. **04**

Q.5 **(a)** Define Homogenous function of two variables x and y of degree n. Also prove the following Euler's theorem for this homogenous function of degree n. **03**
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu .$$

(b) If $u = \tan^{-1}\left[\dfrac{x^3 + y^3}{x - y}\right]$ then prove that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$, **04**

(c) **Attempt the following.**

  1) If $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$ and $z = \rho\cos\phi$ then obtain the Jacobean **03**
  $$J = \frac{\partial(x, y, z)}{\partial(\rho, \phi, \theta)} .$$

  2) In a plane triangle ABC find the extreme values of $\cos A \cos B \cos C$. **04**

Q.6

(a) If $u = \sin^{-1}(x - y)$, $x = 3t$, $y = 4t^3$ then show that $\dfrac{du}{dt} = \dfrac{3}{\sqrt{1 - t^2}}$. **03**

(b) If $u = f(r)$ and $r^2 = x^2 + y^2 + z^2$ then show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = f''(r) + \dfrac{2}{r}f'(r)$, **04**

(c) **Attempt the following.**

  1) Find the equations of the tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$. **03**

  2) Find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. **04**

Q.7 **(a)** Evaluate $\displaystyle\iint_R (2x - y^2)\,dA$;over the triangular region R enclosed between the lines **03** $y = -x + 1$, $y = x + 1$ & $y = 3$.

(b) Evaluate the integral $\displaystyle\int_0^2 \int_{y/2}^1 e^{x^2}\,dx\,dy$ by changing the order of integration. **04**

(c) **Attempt the following.**

  1) Use double integral in polar integral to find the area enclosed by three petaled rose $r = \sin 3\theta$. **03**

  2) Use triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ between the planes $z = 1$ and $x + z = 1$. **04**

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