

GUJARAT TECHNOLOGICAL UNIVERSITY**Diploma Engineering - SEMESTER-II • EXAMINATION – SUMMER 2013****Subject Code: 3320002****Date: 13-06-2013****Subject Name: Advanced Mathematics (Group-I)****Time: 10:30 am - 01:00 pm****Total Marks: 70****Instructions:**

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options. **14**

- 1 $i + i^2 + i^3 + i^4 = \underline{\hspace{2cm}}$.
(a) 1 (b) i (c) $-i$ (d) 0
- 2 $z + \bar{z} = \underline{\hspace{2cm}}$.
(a) $-2\text{Re}(z)$ (b) $\text{Re}(z)$ (c) $2\text{Re}(z)$ (d) $2i\text{Im}(z)$
- 3 $(1 + i)^{-1} = \underline{\hspace{2cm}}$.
(a) $\frac{1}{2} + \frac{i}{2}$ (b) $\frac{1}{2} - \frac{i}{2}$ (c) $-\frac{1}{2} + \frac{i}{2}$ (d) $-\frac{1}{2} - \frac{i}{2}$
- 4 If $f(x) = 2^x - \log_2 x$ then $f(2) = \underline{\hspace{2cm}}$.
(a) 2 (b) 3 (c) -3 (d) 4
- 5 $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \underline{\hspace{2cm}}$.
(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0
- 6 $\lim_{x \rightarrow 0} \frac{5^x - 1}{x} = \underline{\hspace{2cm}}$.
(a) $\log_5 e$ (b) $\log_e 5$ (c) 1 (d) 0
- 7 If $f(x) = e^{2x}$ then $f'(x) = \underline{\hspace{2cm}}$.
(a) $2e$ (b) 2 (c) 1 (d) 0
- 8 $\frac{d}{dx} \log \cos x = \underline{\hspace{2cm}}$.
(a) $\tan x$ (b) $\cot x$ (c) $-\tan x$ (d) $-\cot x$
- 9 $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = \underline{\hspace{2cm}}$.
(a) $\frac{\pi}{2}$ (b) 1 (c) 0 (d) -1
- 10 $\frac{d}{dx} x^x = \underline{\hspace{2cm}}$.
(a) $x - \log x$ (b) $x + \log x$ (c) $x^x(1 + \log x)$ (d) $x \cdot x^{x-1}$
- 11 $\int \tan^2 x \, dx = \underline{\hspace{2cm}}$.

- (a) $2 \tan x \sec^2 x$ (b) $\tan x - x$ (c) $\tan x + x$ (d) $\sec x \tan x$

12 $\int_0^1 \frac{4}{1+x^2} dx = \underline{\hspace{2cm}}$.

- (a) π (b) 2π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

13 The order of the differential equation $x \frac{d^2 y}{dx^2} - 5 \left(\frac{dy}{dx} \right)^3 - 2y = 14$ is $\underline{\hspace{2cm}}$.

- (a) 3 (b) 2 (c) 1 (d) 0

14 The degree of the differential equation $x^2 \frac{dy}{dx} + \sin \left(\frac{d^2 y}{dx^2} \right) = 0$ is $\underline{\hspace{2cm}}$.

- (a) 3 (b) 2 (c) 1 (d) not defined

Q.2 (a) Attempt any two

06

1. Simplify: $\frac{(\cos 2\theta + i \sin 2\theta)^{-3} (\cos 3\theta - i \sin 3\theta)^2}{(\cos 2\theta - i \sin 2\theta)^{-7} (\cos 5\theta - i \sin 5\theta)^3}$

2. Find the modulus and principal argument of $z = \sqrt{3} + i$ and express z into polar form.

3. If $f(x) = \frac{1+x}{1-x}$ then prove that $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$

(b) Attempt any two

08

1. Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{25+x} - 5}{x}$

2. Evaluate: $\lim_{x \rightarrow 0} \frac{\sin 7x - \sin 3x}{\sin x}$

3. If $z = x + iy$ and $|3z| = |z - 4|$ then prove that $x^2 + y^2 + x = 2$

Q.3 (a) Attempt any two

06

1. Find the derivative of $x^2 + 2x - 1$ using definition.

2. If $y = e^{3x} \cos 2x$ then find $\frac{dy}{dx}$.

3. Equation of motion of a particle is $s = t^3 - 6t^2 + 8t - 4$. Then find the velocity and acceleration of the moving particle at $t = 3$ second.

(b) Attempt any two

08

1. If $x = \frac{1}{2} \left(t + \frac{1}{t} \right)$ and $y = \frac{1}{2} \left(t - \frac{1}{t} \right)$ then find $\frac{dy}{dx}$.

2. If $y = e^{\tan^{-1} x}$ then prove that $(1+x^2)y_2 + (2x-1)y_1 = 0$

3. Find the maximum and minimum values of $f(x) = 3x^3 - 4x^2 - x + 5$

Q.4 (a) Attempt any two

06

1. Find $\int \cos x \cdot \sqrt{\sin x} dx$

2. Find $\int \frac{x}{(x+1)(x+2)} dx$

3. Evaluate: $\int_0^{\frac{\pi}{2}} \log \cot x \, dx$

(b) Attempt any two

08

1. Find $\int x \cdot e^{3x} \, dx$

2. Evaluate: $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} \, dx$

3. Find the area bounded by the curve $y = x^2 - 7x + 10$ and x-axis .

Q.5 (a) Attempt any two

06

1. Evaluate: $\lim_{x \rightarrow 0} \frac{15^x - 5^x - 3^x + 1}{x^2}$

2. Find the differential equation for $y = a \sin(x+b)$, where a and b are arbitrary constants.

3. Solve: $\frac{dy}{dx} + y \tan x = \sec^2 x$

(b) Attempt any two

08

Solve: $(x^2 + y^2) dy = 2xy \, dx$

1.

2. Solve: $(x+y+1)^2 \frac{dy}{dx} = 1$

3. Solve: $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. where $y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

Q.1

આપેલા વિકલ્પો માંથી યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો :

14

1 $i + i^2 + i^3 + i^4 = \underline{\hspace{2cm}}$.

(a) 1 (b) i (c) $-i$ (d) 0

2 $z + \bar{z} = \underline{\hspace{2cm}}$.

(a) $-2\text{Re}(z)$ (b) $\text{Re}(z)$ (c) $2\text{Re}(z)$ (d) $2i\text{Im}(z)$

3 $(1+i)^{-1} = \underline{\hspace{2cm}}$.

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4 જો $f(x) = 2^x - \log_2 x$ તો $f(2) = \underline{\hspace{2cm}}$.

(a) 2 (b) 3 (c) -3 (d) 4

5 $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \underline{\hspace{2cm}}$.

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0

6 $\lim_{x \rightarrow 0} \frac{5^x - 1}{x} = \underline{\hspace{2cm}}$.

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- 7 જો $f(x) = e^{2x}$ તો $f'(x) = \underline{\hspace{2cm}}$.
 (a) $2e$ (b) 2 (c) 1 (d) 0
- 8 $\frac{d}{dx} \log \cos x = \underline{\hspace{2cm}}$.
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- 9 $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = \underline{\hspace{2cm}}$.
 (a) $\frac{\pi}{2}$ (b) 1 (c) 0 (d) -1
- 10 $\frac{d}{dx} x^x = \underline{\hspace{2cm}}$.
 (a) $x - \log x$ (b) $x + \log x$ (c) $x^x(1 + \log x)$ (d) $x \cdot x^{x-1}$
- 11 $\int \tan^2 x dx = \underline{\hspace{2cm}}$.
 (a) $2 \tan x \sec^2 x$ (b) $\tan x - x$ (c) $\tan x + x$ (d) $\sec x \tan x$
- 12 $\int_0^1 \frac{4}{1+x^2} dx = \underline{\hspace{2cm}}$.
 (a) π (b) 2π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
- 13 વિકલ સમીકરણ $x \frac{d^2 y}{dx^2} - 5 \left(\frac{dy}{dx} \right)^3 - 2y = 14$ ની કક્ષા $\underline{\hspace{2cm}}$ છે .
 (a) 3 (b) 2 (c) 1 (d) 0
- 14 વિકલ સમીકરણ $x^2 \frac{dy}{dx} + \sin \left(\frac{d^2 y}{dx^2} \right) = 0$ નું પરિમાણ $\underline{\hspace{2cm}}$ છે .
 (a) 3 (b) 2 (c) 1 (d) અવ્યાખ્યાયિત

Q.2 (a) કોઈ પણ બે ગણો:

06

- સાદુરૂપ આપો : $\frac{(\cos 2\theta + i \sin 2\theta)^{-3} (\cos 3\theta - i \sin 3\theta)^2}{(\cos 2\theta - i \sin 2\theta)^{-7} (\cos 5\theta - i \sin 5\theta)^3}$
- $z = \sqrt{3} + i$ નો માનાંક અને કોણાંક શોધો, તથા z ને ધ્રુવિય સ્વરૂપમાં અભિવ્યક્ત કરો
- જો $f(x) = \frac{1+x}{1-x}$ તો સાબિત કરો કે $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$

(b) કોઈ પણ બે ગણો:

08

- કિંમત શોધો : $\lim_{x \rightarrow 0} \frac{\sqrt{25+x} - 5}{x}$
- કિંમત શોધો: $\lim_{x \rightarrow 0} \frac{\sin 7x - \sin 3x}{\sin x}$
- જો $z = x + iy$ અને $|3z| = |z-4|$ તો સાબિત કરો કે $x^2 + y^2 + x = 2$

Q.3 (a) કોઈ પણ બે ગણો:

06

- વ્યાખ્યા ની મદદથી $x^2 + 2x - 1$ નું વિકલિત મેળવો .

2. જો $y = e^{3x} \cos 2x$ તો $\frac{dy}{dx}$ મેળવો.
3. કણની ગતિનું સમીકરણ $s = t^3 - 6t^2 + 8t - 4$ છે .તો $t = 3$ સેકન્ડે કણનો વેગ અને પ્રવેગ શોધો.

(b) કોષ પણ બે ગણો:

08

1. જો $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$ અને $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$ તો $\frac{dy}{dx}$ મેળવો.
2. જો $y = e^{\tan^{-1}x}$ તો સાબિત કરો કે $(1+x^2)y_2 + (2x-1)y_1 = 0$
3. $f(x) = 3x^3 - 4x^2 - x + 5$ નાં મહત્તમ અને ન્યૂનતમ મુલ્યો શોધો.

Q.4 (a) કોષ પણ બે ગણો:

06

1. શોધો: $\int \cos x \cdot \sqrt{\sin x} dx$
2. શોધો : $\int \frac{x}{(x+1)(x+2)} dx$
3. કિંમત શોધો: $\int_0^{\frac{\pi}{2}} \log \cot x dx$

(b) કોષ પણ બે ગણો:

08

1. શોધો : $\int x \cdot e^{3x} dx$
2. કિંમત શોધો: $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} dx$
3. વક્ર $y = x^2 - 7x + 10$ અને X-અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો .

Q.5 (a) કોષ પણ બે ગણો:

06

1. કિંમત શોધો: $\lim_{x \rightarrow 0} \frac{15^x - 5^x - 3^x + 1}{x^2}$
2. $y = a \sin(x+b)$ માટે વિકલ સમીકરણ મેળવો. જ્યાં a અને b સ્વૈર અચળો છે.
3. ઉકેલો: $\frac{dy}{dx} + y \tan x = \sec^2 x$

(b) કોષ પણ બે ગણો:

08

1. ઉકેલો : $(x^2 + y^2) dy = 2xy dx$
2. ઉકેલો : $(x + y + 1)^2 \frac{dy}{dx} = 1$
3. ઉકેલો : $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. જ્યાં $y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$
