

GUJARAT TECHNOLOGICAL UNIVERSITY**Diploma Engineering - SEMESTER-I • EXAMINATION – SUMMER • 2014****Subject Code: 310034****Date: 08-07-2014****Subject Name: Mathematics-I****Time: 10:30 am - 01:00 pm****Total Marks: 70****Instructions:**

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1

Fill in the blanks using appropriate choice from the given options

14

- (1) $\log_2 128 = \dots\dots\dots$
 (i)6 (ii)7 (iii)5 (iv)4
- (2) $\log\left(\frac{m}{n}\right) = \dots\dots\dots$
 (i) $\log m + \log n$ (ii) $\frac{\log m}{\log n}$ (iii) $\log m - \log n$ (iv) $\log n - \log m$
- (3) If 2, x and 8 are in geometric progression then $x = \dots\dots\dots$
 (i)4 (ii) $\frac{1}{4}$ (iii)5 (iv) $\frac{1}{5}$
- (4) nth term of a G.P. 3, 9, 27, =
 (i) 3^n (ii) 3^{n+1} (iii) 3^{n-1} (iv) $3n$
- (5) No. of terms in the expansion of $\left(x + \frac{2}{x}\right)^{11} = \dots\dots$
 (i)11 (ii)12 (iii)10 (iv)13
- (6) 5 th term of $\left(\frac{a}{b} + \frac{b}{a}\right)^5 = \dots\dots\dots$
 (i) $\frac{5b^3}{a^3}$ (ii) $5a^3b^3$ (iii) $\frac{5b^3}{4a^3}$ (iv) $\frac{5}{4}a^3b^3$
- (7) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^T = \dots\dots\dots$
 (i) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
- (8) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \dots\dots\dots$
 (i) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$
- (9) $A^{-1} = \dots\dots\dots$
 (i) $\frac{adjA}{A}$ (ii) $A adjA$ (iii) $\frac{adjA}{|A|}$ (iv) $\frac{|adjA|}{A}$
- (10) If $A = \begin{pmatrix} 3 & 4 \end{pmatrix}$ then $|A| = \dots\dots\dots$
 (i)5 (ii)7 (iii) $\sqrt{5}$ (iv) $\sqrt{7}$
- (11) If $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ then $A.B = \dots\dots\dots$
 (i) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ (ii)6 (iii) $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$ (iv) $\sqrt{6}$
- (12) Unit vector along $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} = \dots\dots\dots$
 (i) $(\sqrt{3} \ \sqrt{3} \ \sqrt{3})$ (ii) $\begin{pmatrix} 3 & 3 & 3 \end{pmatrix}$ (iii) $\left(\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right)$ (iv) $\left(\frac{1}{\sqrt{3}} \ \frac{1}{\sqrt{3}} \ \frac{1}{\sqrt{3}}\right)$
- (13) Period of $\cos(3x + 4) = \dots\dots\dots$
 (i) $\frac{2\pi}{3}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi}{4}$
- (14) $\tan(\pi + \theta) = \dots\dots\dots$
 (i) $\tan \theta$ (ii) $-\tan \theta$ (iii) $\cot \theta$ (iv) $-\cot \theta$

- Q.2 (a) Attempt Any Two 06
- (i) If $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ then prove that
- $$\frac{a}{b} + \frac{b}{a} = 6$$
- (ii) For a geometric progression $T_3 = 9$ and $T_5 = 81$ find a and r
- (iii) Prove that $\frac{1}{\log_2 6} + \frac{1}{\log_3 6} + 1 = 2$

- (b) Attempt Any Two 08
- (i) Find $5 + 55 + 555 + \dots$ up to first n terms
- (ii) If $2 + 4 + 8 + \dots$ up to first n terms = 128, find n
- (iii) Prove that $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1}) = 0$

- Q.3 (a) Attempt Any Two 06
- (i) Find T_6 in the expansion of $\left(\frac{a}{x} + \frac{x}{a}\right)^{10}$

(ii) Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^8$

(iii) $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ and

$$C = \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix} \text{ then find } 2A - 4B + C$$

- (b) Attempt Any Two 08
- (i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ prove that $A^2 - 5A - 2I = 0$
- (ii) Find the inverse of $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$
- (iii) Find the approximate value of $\sqrt{101}$ using Binomial

Theorem

- Q.4 (a) Attempt Any Two 06
- (i) If $a = (2 \ 1 \ -1)$ and $b = (1 \ -1 \ 2)$ and $c = (1 \ -2 \ 1)$ then find the direction cosines of $a + b - 2c$
- (ii) If $a = (1 \ -1 \ 1), b = (2 \ -1 \ 1)$ and $c = (1 \ 1 \ -2)$ then find $a \cdot (b + c)$
- (iii) Find x if the vectors $(2 \ x \ 1)$ and $(4 \ -2 \ 2)$ are perpendicular to each other

- (b) Attempt Any Two 08
- (i) If $a = (2 \ -1 \ 0)$ and $b = (1 \ 3 \ -2)$ then find $|(a + b) \times (a - b)|$
- (ii) Prove that the angle subtended between the vectors $(1 \ 2 \ 0)$ and $(1 \ 1 \ 3)$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$
- (iii) The constant forces $(3 \ -1 \ 2)$ and $(1 \ 3 \ -2)$ act on a particle. Under the action of these forces the particle moves from the point $(2 \ 3 \ 1)$ to the point $(5 \ 2 \ 1)$. Find the total work done

- Q.5 (a) Attempt Any Two 06
- (i) Prove that $\sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{5\pi}{4}\right) + \sin^2\left(\frac{7\pi}{4}\right) = 2$
- (ii) Find $\sin 15^\circ$
- (iii) Find $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(1)$

- (b) Attempt Any Two 08

- (i) Prove that $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$
- (ii) If $\cos \theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$ then prove that $\cos 2\theta = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$
- (iii) Draw a graph of $y = \sin x$. $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

Q.1

સાચો વિકલ્પ પસંદ કરી ખાલી જગ્યા પુરો.

14

- (1) $\log_2 128 = \dots\dots\dots$
 (i) 6 (ii) 7 (iii) 5 (iv) 4
- (2) $\log\left(\frac{m}{n}\right) = \dots\dots\dots$
 (i) $\log m + \log n$ (ii) $\frac{\log m}{\log n}$ (iii) $\log m - \log n$ (iv) $\log n - \log m$
- (3) જો 2, x અને 8 સમગુણોત્તર શ્રેણી માં હોય તો $x = \dots\dots\dots$
 (i) 4 (ii) $\frac{1}{4}$ (iii) 5 (iv) $\frac{1}{5}$
- (4) સમગુણોત્તર શ્રેણી 3, 9, 27... નું n -મું પદ $= \dots\dots\dots$
 (i) 3^n (ii) 3^{n+1} (iii) 3^{n-1} (iv) $3n$
- (5) $\left(x + \frac{2}{x}\right)^{11}$ નાં વિસ્તરણમાં પદની સંખ્યા $= \dots\dots\dots$
 (i) 11 (ii) 12 (iii) 10 (iv) 13
- (6) $\left(\frac{a}{b} + \frac{b}{a}\right)^5$ નું n -મું પદ $= \dots\dots\dots$
 (i) $\frac{5b^3}{a^3}$ (ii) $5a^3b^3$ (iii) $\frac{5b^3}{4a^3}$ (iv) $\frac{5}{4}a^3b^3$
- (7) જો $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ તો $A^T = \dots\dots\dots$
 (i) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
- (8) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \dots\dots\dots$
 (i) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$
- (9) $A^{-1} = \dots\dots\dots$
 (i) $\frac{adj A}{|A|}$ (ii) $A adj A$ (iii) $\frac{adj A}{|A|}$ (iv) $\frac{|adj A|}{A}$
- (10) જો $A = \begin{pmatrix} 3 & 4 \end{pmatrix}$ તો $|A| = \dots\dots\dots$
 (i) 5 (ii) 7 (iii) $\sqrt{5}$ (iv) $\sqrt{7}$
- (11) જો $A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ તો $A \cdot B = \dots\dots\dots$
 (i) $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$ (ii) 6 (iii) $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$ (iv) $\sqrt{6}$
- (12) $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$ ની દિશામાં એકમ સદિશ $= \dots\dots\dots$
 (i) $(\sqrt{3} \ \sqrt{3} \ \sqrt{3})$ (ii) $(3 \ 3 \ 3)$ (iii) $\left(\frac{1}{3} \ \frac{1}{3} \ \frac{1}{3}\right)$ (iv) $\left(\frac{1}{\sqrt{3}} \ \frac{1}{\sqrt{3}} \ \frac{1}{\sqrt{3}}\right)$
- (13) $\cos(3x + 4)$ નું આવર્ત માન $= \dots\dots\dots$
 (i) $\frac{2\pi}{3}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{\pi}{2}$ (iv) $\frac{\pi}{4}$
- (14) $\tan(\pi + \theta) = \dots\dots\dots$
 (i) $\tan \theta$ (ii) $-\tan \theta$ (iii) $\cot \theta$ (iv) $-\cot \theta$

Q.2 (અ) કોઈપણ બે ગણો 06

(i) જો $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$ તો સાબિત કરો કે

$$\frac{a}{b} + \frac{b}{a} = 6$$

(ii) સમગુણોત્તાર શ્રેણી માટે $T_3 = 9$ and $T_5 = 81$
a અને r શોધો.

(iii) સાબિત કરો કે $\frac{1}{\log_2 6} + \frac{1}{\log_3 6} + 1 = 2$

(b) કોઈપણ બે ગણો 08

(i) $5 + 55 + 555 + \dots$ પ્રથમ n-pd સુધી મેળવો

(ii) જો $2 + 4 + 8 + \dots$ પ્રથમ n-pd સુધી $= 128$, તો n શોધો

(iii) સાબિત કરો કે $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1}) = 0$

Q.3 (અ) કોઈપણ બે ગણો 06

(i) $\left(\frac{a}{x} + \frac{x}{a}\right)^{10}$ નાં વિસ્તરણમાં T_6 મેળવો

(ii) $\left(x + \frac{1}{x}\right)^8$ નાં વિસ્તરણમાં મધ્યમ પદ મેળવો

(iii) $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 0 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$ અને

$$C = \begin{bmatrix} 3 & 0 & 5 \\ 6 & 9 & -1 \\ 7 & 8 & -2 \end{bmatrix} \quad 2A-4B+C \text{ મેળવો}$$

(બ) કોઈપણ બે ગણો 08

(i) જો $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ તો સાબિત કરો કે $A^2 - 5A - 2I = 0$

(ii) $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ નો વ્યસ્ત શ્રેણિક મેળવો

(iii) દ્વિ પદી પ્રમે યનો ઉપયોગ કરી $\sqrt{101}$ નુ આસન્ન મુલ્ય મેળવો

Q.4 (અ) કોઈપણ બે ગણો 06

(i) જો $a = (2 \ 1 \ -1)$ $b = (1 \ -1 \ 2)$ અને

$c = (1 \ -2 \ 1)$ તો $a + b - 2c$ નાં દિક્કોસાઇન મેળવો

(ii) જો $a = (1 \ -1 \ 1)$, $b = (2 \ -1 \ 1)$ તો

$c = (1 \ 1 \ -2)$ તો $a \cdot (b + c)$ મેળવો

(iii) જો $(2 \ x \ 1)$ અને $(4 \ -2 \ 2)$ પરસ્પર લંબ હોય તો
x શોધો

(

બ) કોઈપણ બે ગણો

08

- (i) જો $a = (2 \ -1 \ 0)$ and $b = (1 \ 3 \ -2)$ તો
 $|(a + b) \times (a - b)|$ મેળવો
(ii) સાબિત કરો કે

$(1 \ 2 \ 0)$ અને $(1 \ 1 \ 3)$ વચ્ચે અંતરાયે લો ખુણો $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$ છે.

(iii) અચળબળો

$(3 \ -1 \ 2)$ અને $(1 \ 3 \ -2)$ એ ક કણ ઉપર કાર્ય કરે છે.
આ બળોની અસર તે કણ બિંદુ $(2 \ 3 \ 1)$ થી ખસીને
બિંદુ $(5 \ 2 \ 1)$ આગળ આવે છે. કાર્ય શોધો.

Q.5

(અ) કોઈપણ બે ગણો

06

- (i) સાબિત કરો કે $\sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2\left(\frac{5\pi}{4}\right) + \sin^2\left(\frac{7\pi}{4}\right) = 2$
(ii) $\sin 15^\circ$ મેળવો
(iii) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(1)$ મેળવો

(બ) કોઈપણ બે ગણો

08

- (i) સાબિત કરો કે $\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} = 0$
(ii) જો $\cos \theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$ તો
સાબિત કરો કે $\cos 2\theta = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$
(iii) $y = \sin x \cdot \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ નો ગ્રાફ દોરો
