

GUJARAT TECHNOLOGICAL UNIVERSITY
Diploma Engineering - SEMESTER – I • EXAMINATION – WINTER 2012

Subject code: 310034

Date: 08/01/2013

Subject Name: Mathematics-I

Time: 02.30 pm - 05.00 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. English version is considered to be Authentic

- Q.1** (a) Fill in the Blanks **10**
- (1) $\log_2 8 =$ _____
 - (2) If $\log x + \log 2x = \log 18$ then $x =$ _____
 - (3) The geometric mean of 3 and 12 = _____
 - (4) If 0.1, 1 and c are in G.P. then $c =$ _____
 - (5) If ${}^5C_3 = {}^5C_r$ and $r \neq 3$ then $r =$ _____
 - (6) No of terms in the expansion of $\left(x + \frac{1}{x}\right)^5 =$ _____
 - (7) $\sin^2 \theta + \cos^2 \theta =$ _____
 - (8) $\sin(180^\circ + \theta) =$ _____
 - (9) $\sin^{-1} x +$ _____ $= \frac{\pi}{2}$
 - (10) Period of $\sin(2x+3) =$ _____
- (b) Prove that $\frac{1}{\log_2 6} + \frac{1}{\log_3 6} = 1$ **02**
- (c) For G.P. 1,3,9,..... if $T_n = 2187$ then find n. **02**
- Q.2** (a) (1) Prove that $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \log_{a^4} x^4 = 4 \log_a x$ **04**
- (2) Prove that $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$ **03**
- (b) (1) Find the sum of first n terms of $3+33+333+\dots$ **04**
- (2) Find three geometric mean between 6 and 486 **03**
- Q.3** (a) (1) Find the coefficient of x^6 in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ **04**
- (2) Find the middle term in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^8$ **03**
- (b) (1) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I = 0$. **04**

(2) If $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 0 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 1 & 2 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$ then find $2A-B+C$. 03

Q.4 (a) (1) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ then find A^{-1} . 04

(2) Solve the given equations using matrix method, $3x-y=1$ and $x+2y=5$. 03

(b) (1) A particle moves from $(-1,2,1)$ to $(2,3,-1)$ under the effect of the forces $(1,2,1)$ and $(2,-1,0)$. Find the work done. 04

(2) If $\vec{a} = (1,1,1)$, $\vec{b} = (2,1,2)$, $\vec{c} = (-1,0,3)$ then find $|2\vec{a} + \vec{b} - \vec{c}|$ 03

Q.5 (a) (1) Find unit vector perpendicular to $\vec{a} = (3,1,2)$ and $\vec{b} = (2,-2,4)$ 04

(2) Find the angle between the vectors $\vec{a} = (1,2,4)$ and $\vec{b} = (3,1,2)$ 03

(b) (1) Prove that $\cot \frac{\pi}{20} \cot \frac{3\pi}{20} \cot \frac{5\pi}{20} \cot \frac{7\pi}{20} \cot \frac{9\pi}{20} = 1$ 04

(2) prove that $\frac{\sec \theta + 2 \sin \theta}{\sin \theta + \cos \theta} = 1 + \tan \theta$ 03

Q. 6 (a) (1) Draw the graph of $y=\sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ 04

(2) Simplify $\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi-\theta)} + \frac{\tan\left(\frac{\pi}{2}+\theta\right)}{\cot(\pi+\theta)} + \frac{\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)}{\sec(\pi+\theta)}$ 03

(b) (1) Prove that $\cos A \sin(B-C) + \cos B \sin(C-A) + \cos C \sin(A-B) = 0$ 04

(2) Prove that $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$ 03

Q. 7 (a) (1) Draw the graph of $y=\cos x$, $0 \leq x \leq \pi$ 04

(2) Prove that $\frac{\cos A + \cos 3A + \cos 5A}{\sin A + \sin 3A + \sin 5A} = \cot 3A$ 03

(b) (1) For acute angled ΔABC prove sine rule in standard notations. 04

(2) Prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ 03

Q.1 (a) ખાલી જગ્યાપૂરો**10**

- (1) $\log_2 8 = \underline{\hspace{2cm}}$
- (2) જો $\log x + \log 2x = \log 18$ તો $x = \underline{\hspace{2cm}}$
- (3) 3 અને 12 નો સમગુણોતર મધ્યક = $\underline{\hspace{2cm}}$
- (4) જો 0.1, 1 અને c G.P. મા હોય તો $c = \underline{\hspace{2cm}}$
- (5) જો ${}^5C_3 = 5C_r$, અને $r \neq 3$ તો $r = \underline{\hspace{2cm}}$
- (6) $\left(x + \frac{1}{x}\right)^5$ ના વિસ્તરણમાં કુલ પદની સંખ્યા = $\underline{\hspace{2cm}}$
- (7) $\sin^2 \theta + \cos^2 \theta = \underline{\hspace{2cm}}$
- (8) $\sin(180^\circ + \theta) = \underline{\hspace{2cm}}$
- (9) $\sin^{-1} x + \underline{\hspace{2cm}} = \frac{\pi}{2}$
- (10) $\sin(2x+3)$ નું આવર્તમાન = $\underline{\hspace{2cm}}$

(b) સાબિત કરો કે $\frac{1}{\log_2 6} + \frac{1}{\log_3 6} = 1$

02

(c) G.P. 1,3,9,..... માટે $T_n = 2187$ તો n શોધો.

02**Q.2 (a) (1) સાબિત કરો કે $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + \log_{a^4} x^4 = 4 \log_a x$** **04**

(2) સાબિત કરો કે $\frac{1}{\log_6 24} + \frac{1}{\log_{12} 24} + \frac{1}{\log_8 24} = 2$

03

(b) (1) $3+33+333+\dots$ ના પ્રથમ n પદોનો સરવાળો શોધો.

04

(2) 6 અને 486 વચ્ચે ત્રણ સમગુણોતર મધ્યક મેળવો.

03**Q.3 (a) (1) $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ ના વિસ્તરણમાં x^6 નો સહગુણક શોધો.****04**

(2) $\left(\frac{x^2}{2} - \frac{2}{x}\right)^8$ ના વિસ્તરણનું મધ્યમપદ શોધો.

03

(b) (1) જો $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ તો બતાવો કે $A^2 - 4A - 5I = 0$.

04

(2) જો $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 2 \\ 0 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 & 2 \\ 0 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}$ તો

03

$2A - B + C$ શોધો..

- Q.4** (a) (1) જો $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ તો A^{-1} શોધો. **04**
- (2) આપેલા સમીકરણ નો શ્રેણિકની મદદથી ઉકેલ શોધો, $3x-y=1$ અને $x+2y=5$. **03**
- (b) (1) બંને $(1,2,1)$ અને $(2,-1,0)$ ની અસર તળે એક કણ નુ બિન્દુ $(-1,2,1)$ થી બિન્દુ $(2,3,-1)$ સુધી સ્થાનાંતર થાય છે. થયેલ કાર્ય શોધો. **04**
- (2) જો $\vec{a} = (1,1,1)$, $\vec{b} = (2,1,2)$, $\vec{c} = (-1,0,3)$ તો $|2\vec{a} + \vec{b} - \vec{c}|$ શોધો **03**
- Q.5** (a) (1) $\vec{a} = (3,1,2)$ અને $\vec{b} = (2,-2,4)$ ને એકમ લંબ સદિશ શોધો. **04**
- (2) $\vec{a} = (1,2,4)$ અને $\vec{b} = (3,1,2)$ વચ્ચે નો ખૂણો શોધો. **03**
- (b) (1) સાબિત કરો કે $\cot \frac{\pi}{20} \cot \frac{3\pi}{20} \cot \frac{5\pi}{20} \cot \frac{7\pi}{20} \cot \frac{9\pi}{20} = 1$ **04**
- (2) સાબિત કરો કે $\frac{\sec \theta + 2 \sin \theta}{\sin \theta + \cos \theta} = 1 + \tan \theta$ **03**
- Q. 6** (a) (1) $y = \sin x$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ નો આલેખ દોરો. **04**
- (2) સાદુરુપ આપો $\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos(\pi - \theta)} + \frac{\tan\left(\frac{\pi}{2} + \theta\right)}{\cot(\pi + \theta)} + \frac{\operatorname{cosec}\left(\frac{\pi}{2} + \theta\right)}{\sec(\pi + \theta)}$ **03**
- (b) (1) સાબિત કરો કે $\cos A \sin(B-C) + \cos B \sin(C-A) + \cos C \sin(A-B) = 0$ **04**
- (2) સાબિત કરો કે $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A} = \tan A$ **03**
- Q. 7** (a) (1) $y = \cos x$, $0 \leq x \leq \pi$ નો આલેખ દોરો. **04**
- (2) સાબિત કરો કે $\frac{\cos A + \cos 3A + \cos 5A}{\sin A + \sin 3A + \sin 5A} = \cot 3A$ **03**
- (b) (1) લઘુકોણ ΔABC માટે પ્રમાણિત સંકેતમા \sin નો નિયમ સાબિત કરો. **04**
- (2) સાબિત કરો કે $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{2}\right)$ **03**
