Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

DIPLOMA ENGINEERING – SEMESTER I• EXAMINATION – WINTER-2015

Subject Code: 310034 **Date: 12/01/2016**

Subject Name: Mathematics-I

Time: 10:30 AM to 1:00PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of programmable & Communication aids are strictly prohibited.**
5. **Use of only simple calculator is permitted in Mathematics.**
6. **English version is authentic.**

Q.1 Fill in the blanks . **14**

1. $\log_9(9)$ = _____

   (a) -1 (b) 0 (c) 1 (d) 2

2. $\log_{11}(1)$ = _____

   (a) -1 (b) 0 (c) 1 (d) 2

3. If 3,x,27 are in geometric progression then x= _____

   (a) 3 (b) -3 (c) 6 (d) 9

4. 7th term of the sequence 5,10,20, ------- is _____

   (a) 320 (b) 160 (c) 640 (d) 80

5. ${}^5C_3$ = _____

   (a) 5 (b) 10 (c) 15 (d) 20

6. Number of terms in the expansion of $(2x-5y)^5$ is _____

   (a) 3 (b) 4 (c) 5 (d) 6

7. $|i+2j-3k|$ = _____

   (a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{15}$ (d) $\sqrt{16}$

8. If $\bar{x}$ =(-1,1,0) and $\bar{y}$ =(3,2,1) then $\bar{x}\cdot\bar{y}$ = _____

   (a) -1 (b) 0 (c) 1 (d) 2

9. $\text{Sin}^{-1}(\frac{1}{2})$ = _____

   (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

10. $\tan 30^\circ$ = _____

    (a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

11. 540° = _____ radian

    (a) 3π (b) 4π (c) 5π (d) 6π

12. $\begin{vmatrix} 5 & -3 \\ 1 & -1 \end{vmatrix}$ = _____

    (a) -1 (b) -2 (c) 1 (d) 2

13. If A= $\begin{bmatrix} 4 & -3 \\ 5 & -8 \end{bmatrix}$ then adj(A)= _____

(a) $\begin{bmatrix} 4 & 5 \\ -3 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -8 & -3 \\ 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 3 \\ -5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 3 \\ -5 & 4 \end{bmatrix}$

14. If A= $\begin{bmatrix} 4 & -3 \\ 5 & -8 \end{bmatrix}$ then A^T= _____

(a) $\begin{bmatrix} 4 & 5 \\ -3 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -8 & -3 \\ 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 3 \\ -5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 3 \\ -5 & 4 \end{bmatrix}$

Q.2 (a) Attempt any two. **06**

1. Prove that $\frac{1}{\log_6(24)} + \frac{1}{\log_{12}(24)} + \frac{1}{\log_8(24)} = 2$
2. Prove that $\log(x+\sqrt{x^2-1}) + \log(x-\sqrt{x^2-1}) = 0$
3. If for a G.P. T_8=243 and T_5=9, then find a and r .

(b) Attempt any two. **08**

1. If $\log(\frac{a+b}{2}) = \frac{1}{2}$ (loga+logb) then prove that a=b.
2. Find the sum of the series 7+77+777+_ _ _ upto n terms.
3. Find the approximate value of $\sqrt{98}$ using Binomial Theorem.

Q.3 (a) Attempt any two. **06**

1. If A= $\begin{bmatrix} -1 & -4 \\ 3 & -5 \end{bmatrix}$, then find A^{-1}.
2. If A= $\begin{bmatrix} 3 & 9 \\ 4 & -6 \end{bmatrix}$ and B= $\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}$, then find 3A-2B .
3. Find the middle term in the expansion of $(\frac{x}{2} - \frac{2}{y})^8$.

(b) Attempt any two. **08**

1. If A= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then show that A^2-5A+7I=0,where I is the unit matrix.
2. If A= $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ then find A^{-1} .
3. Find the constant term in the expansion of $(2x^2 - \frac{1}{x})^6$.

Q.4 (a) Attempt any two. **06**

1. Find $|(2i-3j+k)\times(i-j+2k)|$.
2. If $\bar{a}$ =(3,1,-1), $\bar{b}$ =(-2,0,1) and $\bar{c}$ =(1,1,-1) then find $\bar{a}\cdot(\bar{b}+\bar{c})$.
3. Prove that $|\bar{x}+\bar{y}|^2 + |\bar{x}-\bar{y}|^2 = 2|\bar{x}|^2 + 2|\bar{y}|^2$.

(b) Attempt any two. **08**

1. If $\bar{a}$ =5i-3j+2k, $\bar{b}$ =2i+3j-k and $\bar{c}$ =i+2j+3k then find the value of $|2\bar{a}-3\bar{b}+4\bar{c}|$.
2. Find the angle between the vectors 2i+j+4k and i+j+k .
3. A particle moves from the point 3i-2j+k to the point i+3j-4k under the effect of constant forces i-j+k, i+j-3k and 4i+5j-6k. Find the work done.

Q.5 (a) Attempt any two. **06**

1. Prove that $\tan\frac{\pi}{20}\ \tan\frac{3\pi}{20}\ \tan\frac{5\pi}{20}\ \tan\frac{7\pi}{20}\ \tan\frac{9\pi}{20}=1.$

2. Prove that $\tan 55^\circ = \frac{\cos 10^\circ + \sin 10^\circ}{\cos 10^\circ - \sin 10^\circ}$

3. Prove that $\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3}) = \frac{\pi}{4}$

(b) Attempt any two. **08**

1. Prove that $\frac{\tan(\pi-\theta)}{\tan(\pi+\theta)} \times \frac{\cot(\pi+\theta)}{\cot(\pi-\theta)} \times \frac{\tan(2\pi+\theta)}{\cot(2\pi-\theta)} = -\tan^2\theta$

2. If $\tan\theta = \frac{1}{2}$, then prove that $7\cos 2\theta + 8\sin 2\theta = \frac{53}{5}$.

3. Draw the graph of y=cosx, $0 \le x \le 2\pi$.

ગુજરાતી

પ્રશ્ન. 1 ખાલી જગ્યા પૂરો . **14**

1. $\log_9(9)$ = _____

(a) -1 (b) 0 (c) 1 (d) 2

2. $\log_{11}(1)$ = _____

(a) -1 (b) 0 (c) 1 (d) 2

3. જો 3,x,27 સમગુણોત્તર શ્રેણીમાં હોય તો x= _____

(a) 3 (b) -3 (c) 6 (d) 9

4. શ્રેણી 5,10,20, ------- નું સાતમું પદ _____ થાય .

(a) 320 (b) 160 (c) 640 (d) 80

5. ${}^5C_3$ = _____

(a) 5 (b) 10 (c) 15 (d) 20

6. $(2x-5y)^5$ ના વિસ્તરણમાં કુલ પદોની સંખ્યા _____ થાય.

(a) 3 (b) 4 (c) 5 (d) 6

7. $|i+2j-3k|$ = _____

(a) $\sqrt{13}$ (b) $\sqrt{14}$ (c) $\sqrt{15}$ (d) $\sqrt{16}$

8. જો $\bar{x}$ =(-1,1,0) અને $\bar{y}$ =(3,2,1) તો $\bar{x}\cdot\bar{y}$ = _____

(a) -1 (b) 0 (c) 1 (d) 2

9. $\text{Sin}^{-1}(\frac{1}{2})$ = _____

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

10. $\tan 30^\circ$ = _____

(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

11. 540° = _____ રેડિયન

(a) 3π (b) 4π (c) 5π (d) 6π

12. $\begin{vmatrix} 5 & -3 \\ 1 & -1 \end{vmatrix}$ = _____

(a) -1 (b) -2 (c) 1 (d) 2

13. જો $A=\begin{bmatrix} 4 & -3 \\ 5 & -8 \end{bmatrix}$ તો adj(A)= _____

(a) $\begin{bmatrix} 4 & 5 \\ -3 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -8 & -3 \\ 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 3 \\ -5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 3 \\ -5 & 4 \end{bmatrix}$

14. જો $A=\begin{bmatrix} 4 & -3 \\ 5 & -8 \end{bmatrix}$ તો $A^T=$ _____

(a) $\begin{bmatrix} 4 & 5 \\ -3 & -8 \end{bmatrix}$ (b) $\begin{bmatrix} -8 & -3 \\ 5 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & 3 \\ -5 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 3 \\ -5 & 4 \end{bmatrix}$

પ્રશ્ન. ૨ (અ) કોઇપણ બે ગણો. **06**

1. સાબિત કરો કે $\frac{1}{\log_6(24)}+\frac{1}{\log_{12}(24)}+\frac{1}{\log_8(24)}=2$
2. સાબિત કરો કે $\log(x+\sqrt{x^2-1})+\log(x-\sqrt{x^2-1})=0$
3. જો સમગુણોત્તર શ્રેણીમાં $T_8=243$ અને $T_5=9$, તો a અને r શોધો .

(બ) કોઇપણ બે ગણો. **08**

1. જો $\log(\frac{a+b}{2}) = \frac{1}{2}(\log a+\log b)$ તો સાબિત કરો કે a=b.
2. શ્રેણી 7+77+777+ _ _ _ ના પ્રથમ n પદોનો સરવાળો શોધો.
3. $\sqrt{98}$ નું આસન્ન મૂલ્ય શોધો.

Q.3 (અ) કોઇપણ બે ગણો. **06**

1. જો $A=\begin{bmatrix} -1 & -4 \\ 3 & -5 \end{bmatrix}$ તો A^{-1} શોધો.
2. જો $A=\begin{bmatrix} 3 & 9 \\ 4 & -6 \end{bmatrix}$ અને $B=\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}$ તો 3A-2B ની કિંમત શોધો .
3. $(\frac{x}{2}-\frac{2}{y})^8$ ના વિસ્તરણનું મધ્યમ પદ શોધો.

(બ) કોઇપણ બે ગણો. **08**

1. જો $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, તો સાબિત કરો કે $A^2-5A+7I=0$, જ્યાં I એકમ શ્રેણીક છે .
2. જો $A=\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$ તો A^{-1} શોધો .
3. $(2x^2-\frac{1}{x})^6$ ના વિસ્તરણનું અચળ પદ શોધો.

Q.4 (અ) કોઇપણ બે ગણો. **06**

1. $|(2i-3j+k)\times(i-j+2k)|$ ની કિંમત શોધો .
2. જો $\bar{a}$=(3,1,-1), $\bar{b}$=(-2,0,1) અને $\bar{c}$=(1,1,-1) તો $\bar{a}\cdot(\bar{b}+\bar{c})$ ની કિંમત શોધો .
3. સાબિત કરો કે $|\bar{x}+\bar{y}|^2+|\bar{x}-\bar{y}|^2=2|\bar{x}|^2+2|\bar{y}|^2$.

(બ) કોઇપણ બે ગણો. **08**

1. જો $\bar{a}$ =5i-3j+2k, $\bar{b}$ =2i+3j-k અને $\bar{c}$ =i+2j+3k તો $\left|2\bar{a}-3\bar{b}+4\bar{c}\right|$ ની કિંમત શોધો .
2. સદિશો 2i+j+4k અને i+j+k વચ્ચેનો ખૂણો શોધો .
3. અચળ બળો i-j+k, i+j-3k અને 4i+5j-6k ની અસર નીચે એક કણનું 3i-2j+k થી i+3j-4k સુધી સ્થાનાંતર થાય તો થતું કાર્ય શોધો .

Q.5 (અ) કોઇપણ બે ગણો. **06**

1. સાબિત કરો કે $\tan\frac{\pi}{20}\ \tan\frac{3\pi}{20}\ \tan\frac{5\pi}{20}\ \tan\frac{7\pi}{20}\ \tan\frac{9\pi}{20}=1$
2. સાબિત કરો કે $\tan 55^\circ=\frac{\cos 10^\circ+\sin 10^\circ}{\cos 10^\circ-\sin 10^\circ}$
3. સાબિત કરો કે $\tan^{-1}(\frac{1}{2})+\tan^{-1}(\frac{1}{3})=\frac{\pi}{4}$

(બ) કોઇપણ બે ગણો. **08**

1. સાબિત કરો કે $\frac{\tan(\pi-\theta)}{\tan(\pi+\theta)}\times\frac{\cot(\pi+\theta)}{\cot(\pi-\theta)}\times\frac{\tan(2\pi+\theta)}{\cot(2\pi-\theta)}=-\tan^2\theta$
2. જો $\tan\theta=\frac{1}{2}$ તો સાબિત કરો કે $7\cos 2\theta+8\sin 2\theta=\frac{53}{5}$.
3. y=cosx, $0\le x\le 2\pi$ નો આલેખ દોરો .

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