Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA INTEGRATED- SEMESTER–II• EXAMINATION – SUMMER 2017

Subject Code: 4420601 Date: 31-05-2017

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10.30am to 01.00pm Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define Join-irreducible elements & atom of the Boolean algebra. **07**
Draw Hasse diagram of (S_{210}, D) and determine Join-irreducible elements and atom of the Boolean algebra (S_{210}, D).

(b) For the poset <{{1},{2}, {4}, {1,2}, {1,4}, {2,4}, {3,4}, {1,3,4}, {2,3,4}}, $\subseteq$> **07**
Draw the Hasse diagram and find :
1) maximal elements and minimal elements
2) Greatest element and least element, if exists
3) Lower bounds of {1,3,4} and {2,3,4}
4) Upper bounds of {2,4} and {3,4}

Q.2 **(a)** (i) Show that in a lattice if $x \le y \le z$ then $x \oplus y = y * z$ **03**

(ii) $(a * b) \oplus (a * b') = a$ **02**

(iii) $a * (a' \oplus b) = a * b$ **02**

(b) Define Sublattice. **07**
Check whether the following are sublattice of (S_{30}, GCD,LCM) or not? Justify.
(i)({1,2,3,6} , GCD,LCM)
(ii)({1,3,6,15}, GCD,LCM)

OR

(b) Let $(L, \le)$ be a lattice. For any a,b $\in$ L prove that $a \le b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **07**

Q.3 **(a)** Use the Quine - McCluskey method to simplify the sum-of-products expression: **07**
f(a,b,c,d) = $\sum$(10,12,13,14,15)

(b) Define: Boolean Algebra. Find all Sub Boolean algebra of Boolean algebra **07**
‹ S_{30}, Λ, V, ' , 0 , 1 ›

OR

Q.3 **(a)** Check whether (S_{45} , D) is a complemented lattice or not? Justify. **07**

(b) Use Karnaugh map representation to find a minimal sum-of-products expression of the following: **07**
(i)f(*a*,*b*,*c*,*d*)=Σ(5,7,10,13,15)
(ii) *f*(*a*,*b*,*c*,*d*)=Σ(0,1,2,3,13,15)

Q.4 **(a)** (i) Define:Group. Show that in a group (G,*), **04**
for any a,b $\in$ G if $(a*b)^2 = a^2 * b^2$ then (G,*) must be abelian.

(ii)If <G, *> be a group then for any two elements a and b of <G, *>, prove that $(a*b)^{-1} = b^{-1} * a^{-1}$ **03**

(b) Define subgroup of a group, left coset of a subgroup <H, *> in the group <G, *>. Find left cosets of {[0], [3]} in the group <Z_6, +6>. **07**

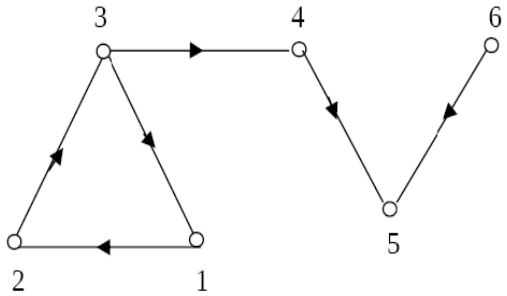
OR

Q.4 **(a)** Define Cyclic group. **07**
Show that $(Z_4,+_4)$ is a cyclic group and find all the subgroups of $(Z_4,+_4)$.

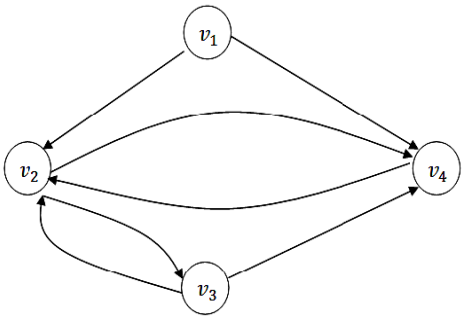
(b) Define "Group" "normal subgroup" of a group. Determine all the subgroups of the symmetric group $<S_3, \lozenge>$ given in the table below. **07**

$\lozenge$	P1	P2	P3	P4	P5	P6
P1	P1	P2	P3	P4	P5	P6
P2	P2	P1	P5	P6	P3	P4
P3	P3	P6	P1	P5	P4	P2
P4	P4	P5	P6	P1	P2	P3
P5	P5	P4	P2	P3	P6	P1
P6	P6	P3	P4	P2	P1	P5

Q.5 **(a)** Define Nodebase. Find Nodebase of the following digraph. **07**

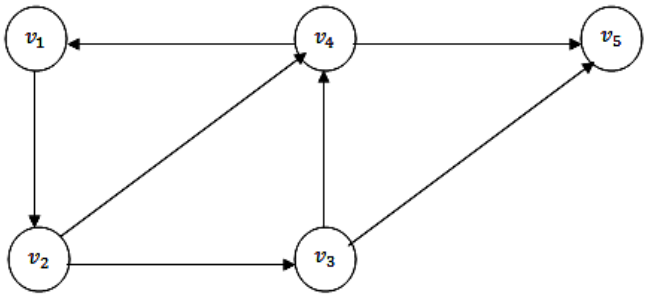


(b) Define Adjacency matrix. **07**
Obtain the adjacency matrix *A* of the digraph given below. Also find the elementary paths of lengths 1 and 2 from to v_1 to v_4.



OR

Q.5 **(a)** Define: Strong component, Unilateral component and weak component of the digraph. **07**
Determine Strong component, Unilateral component and weak component of the following digraph.



(b) Define Binary tree. Convert the given tree into binary tree **07**
(v0(v1(v2)(v3)(v4))(v5(v6)(v7)(v8)(v9))(v10(v11)(v12)))

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